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Quadratic Equations and Inequalities

The University of Minnesota wishes to set up a rectangular botanical garden. They have 300 meters of fence to enclose 5,000 square meters for the garden. What are the dimensions of the garden?

6–1 ■ Solution by factoring and extracting roots

Solution by factoring

So far we have discussed linear (first-degree) equations in one variable having at most one solution. We now consider the solution set of a *quadratic* (second-degree) *equation* in one variable that will have at most *two possible solutions*.

Example

In electrical circuits, the flow of current varies with time. In a certain circuit, this relationship is expressed as

$$I = 12 - 12t^2$$

where I is the current in amperes and t is the time in seconds. When will the current flow be 0? To answer this question, it is necessary to set $I = 0$ in the equation. This yields

$$0 = 12 - 12t^2$$

which is a quadratic equation in the variable t.

Definition of a quadratic equation in one variable

A **quadratic equation in one variable** is any second-degree equation that can be written in the form

$$ax^2 + bx + c = 0$$

where a, b, and c are real numbers, $a > 0$. We call this the **standard form** of a quadratic equation in one variable, x.

Note It is necessary to restrict a so that it is not equal to zero. If $a = 0$, then we have

$$0 \cdot x^2 + bx + c = 0 \quad \text{or} \quad bx + c = 0$$

and we have a first-degree (linear) equation.

If $b = 0$ or $c = 0$, then the equation is of the form

$$ax^2 + c = 0 \quad \text{or} \quad ax^2 + bx = 0$$

and we still have a second-degree (quadratic) equation. The following are examples of quadratic equations:

$$2x^2 - x + 5 = 0; \quad 5x^2 + 2x = 0; \quad 4x^2 - 12 = 0$$

When the quadratic expression $ax^2 + bx + c$ can be factored, we use the **zero product property,** stated in section 4–1, to solve the quadratic equation. The following procedure outlines the steps for solving quadratic equations using the zero product property.

To solve a quadratic equation by factoring

1. Write the equation in standard form, if it is not given in this form.
2. Completely factor the quadratic expression.
3. Set each factor containing the variable equal to 0 and solve the resulting linear equations.

▥ Example 6–1 A

Find the solution set of the following quadratic equations.

1. $x^2 + 6 = 7x$

   Since the equation is not in standard form, we subtract $7x$ from each member to obtain the standard form.

   $$
   \begin{aligned}
   x^2 - 7x + 6 &= 0 \\
   (x - 6)(x - 1) &= 0 \qquad &&\text{Factor the left member} \\
   x - 6 = 0 \quad \text{or} \quad x - 1 &= 0 \qquad &&\text{Set each factor equal to 0} \\
   x = 6 \qquad\qquad x &= 1 \qquad &&\text{Solve resulting linear equations}
   \end{aligned}
   $$

   The solution set is $\{1, 6\}$.

   To check our solutions, as we should always do, substitute 6 for x and then 1 for x in the original equation.

When $x = 6$		When $x = 1$	
$x^2 + 6 = 7x$		$x^2 + 6 = 7x$	
$(6)^2 + 6 = 7(6)$	Replace x with 6	$(1)^2 + 6 = 7(1)$	Replace x with 1
$36 + 6 = 42$		$1 + 6 = 7$	
$42 = 42$	(True)	$7 = 7$	(True)

   We will not show a check in the remaining examples, but we should *always* check the solutions.

2. $6p^2 - 5p = 0$

The equation is already in standard form, so we factor the common factor p.

$$p(6p - 5) = 0 \qquad \text{Factor the left member}$$

$p = 0 \quad$ or $\quad 6p - 5 = 0 \qquad$ Set each factor equal to 0

$p = 0 \qquad\qquad 6p = 5 \qquad$ Add 5 to each member

$p = 0 \qquad\qquad p = \dfrac{5}{6} \qquad$ Divide each member by 6

The solution set is $\left\{0, \dfrac{5}{6}\right\}$.

Note A common error is to forget the factor p. That is, the solution $p = 0$ is sometimes omitted by students. Be careful!

3. $3q^2 + 4q + \dfrac{4}{3} = 0$

Since we have a rational equation (an equation containing at least one rational term), we clear the denominator by multiplying all terms in each member of the equation by 3.

$9q^2 + 12q + 4 = 0 \qquad$ Multiply each member by 3

$(3q + 2)^2 = 0 \qquad$ Factor $9q^2 + 12q + 4$

$3q + 2 = 0 \qquad$ Set the factor equal to 0

$3q = -2 \qquad$ Subtract 2 from each member

$q = -\dfrac{2}{3} \qquad$ Divide each member by 3

The solution set is $\left\{-\dfrac{2}{3}\right\}$.

The linear factor $3q + 2$ appears twice. When this occurs, we say that $-\dfrac{2}{3}$ is a solution of *multiplicity two*.

4. $(2x - 1)(x + 2) = -3$

$2x^2 + 3x - 2 = -3 \qquad$ Multiply as indicated

$2x^2 + 3x + 1 = 0 \qquad$ Add 3 to each member

$(2x + 1)(x + 1) = 0 \qquad$ Factor the left member

$2x + 1 = 0 \quad$ or $\quad x + 1 = 0 \qquad$ Set each factor equal to 0

$x = -\dfrac{1}{2} \qquad\qquad x = -1 \qquad$ Solve for x

The solution set is $\left\{-1, -\dfrac{1}{2}\right\}$.

5. The power output P of an 80-volt electric generator is defined by $P = 80I - 5I^2$, where I is the current in amperes. What current I is necessary for the power output of 140 watts?

Given $P = 140$, we substitute to obtain the quadratic equation

$$140 = 80I - 5I^2$$

Add $5I^2 - 80I$ to both members.

$$5I^2 - 80I + 140 = 0 \qquad \text{Write in standard form}$$
$$5(I^2 - 16I + 28) = 0 \qquad \text{Factor the left member}$$
$$5(I - 14)(I - 2) = 0$$
$$I - 14 = 0 \quad \text{or} \quad I - 2 = 0 \qquad \text{Set each factor equal to 0}$$
$$I = 14 \qquad\qquad I = 2$$

The generator will produce 140 watts when $I = 14$ amperes or when $I = 2$ amperes.

Note We do not set 5 equal to 0 since the factor 5 does not contain a variable.

▶ *Quick check* Find the solution set of $3x^2 - 14x = 0$.

Solution by extracting the roots

Consider now the quadratic equation $x^2 - 16 = 0$. Factoring, we obtain

$$(x - 4)(x + 4) = 0$$

Since $x = 4$ when $x - 4 = 0$ and $x = -4$ when $x + 4 = 0$, the solution set is $\{4, -4\}$. We can also solve this equation by writing it in the form $x^2 = 16$. Since 16 is greater than or equal to zero, x is a number that, when squared, yields 16. This can be accomplished when

$$x = \sqrt{16} \quad \text{or} \quad x = -\sqrt{16}$$

since $(\sqrt{16})^2 = 16$ and $(-\sqrt{16})^2 = 16$. Thus the solutions of the quadratic equation $x^2 = 16$ are

$$x = \sqrt{16} = 4 \quad \text{or} \quad x = -\sqrt{16} = -4$$

which are the same values that we determined by factoring.

This example illustrates the square root property.

Square root property

Given real number p and $x^2 = p$, then

$$x = \sqrt{p} \quad \text{or} \quad x = -\sqrt{p}$$

We use this property to solve certain types of quadratic equations by **extracting the roots**.

Note We can write $x = \sqrt{p}$ or $x = -\sqrt{p}$ as $x = \pm\sqrt{p}$, which we read "x equals plus or minus the square root of p."

■ **Example 6–1 B**

Find the solution set by extracting the roots.

1. $x^2 = 13$

$x = \sqrt{13}$ or $x = -\sqrt{13}$ Extract the roots

The solution set is $\{\sqrt{13}, -\sqrt{13}\}$.

Note We can write the solution set as $\{\pm \sqrt{13}\}$, where "$\pm$" is read "plus or minus."

2. $(3x + 2)^2 = 7$

$3x + 2 = \sqrt{7}$ or $3x + 2 = -\sqrt{7}$ Extract the roots
$3x = -2 + \sqrt{7}$ or $3x = -2 - \sqrt{7}$ Add −2 to each member
$x = \dfrac{-2 + \sqrt{7}}{3}$ or $x = \dfrac{-2 - \sqrt{7}}{3}$ Divide each member by 3

The solution set is $\left\{ \dfrac{-2 - \sqrt{7}}{3}, \dfrac{-2 + \sqrt{7}}{3} \right\}$.

3. $(x - 3)^2 = -4$

$x - 3 = \sqrt{-4}$ or $x - 3 = -\sqrt{-4}$ Extract the roots

However $\sqrt{-4}$ is not a real number and the equation has *no real number solution*. We have learned that $\sqrt{-4}$ is equal to $2i$, where $i = \sqrt{-1}$. Then

$x - 3 = 2i$ or $x - 3 = -2i$ $\sqrt{-4} = 2i$
$x = 3 + 2i$ $x = 3 - 2i$ Add 3 to each member

The solution set is $\{3 + 2i, 3 - 2i\}$.

Note When the equation is in the form $(kx + \ell)^2 = p$, do *not* perform the indicated multiplication in the left member.

▶ **Quick check** Find the solution set of $(4z + 5)^2 = 11$.   ■

Many times when we translate a word problem into mathematical language, we obtain a quadratic equation. Following are some examples.
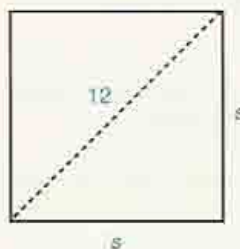
■ **Example 6–1 C**

Solve the following problems by setting up an equation and solving it.

1. Find the length of each side of a square if the diagonal is 12 centimeters long.

Let $s =$ the length of the side of the square. Using the Pythagorean Theorem for right triangles, we obtain the equation
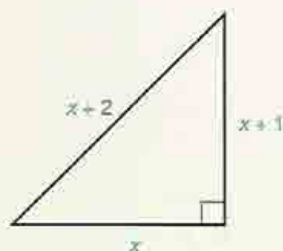
$s^2 + s^2 = (12)^2$
$2s^2 = 144$
$s^2 = 72$

$s = \sqrt{72} = 6\sqrt{2}$ or $s = -\sqrt{72} = -6\sqrt{2}$ Extract the roots

We reject the negative solution since we are finding the length of a side. Thus $s = 6\sqrt{2}$ centimeters.

2. The lengths of the three sides of a right triangle are three consecutive integers. Find the lengths of the three sides.

Let x = the length of the shortest side, then $x + 1$ = the length of the next longer side, and $x + 2$ = the length of the longest side.

$$\underset{\text{(side)}^2}{x^2} \; \underset{\text{plus}}{+} \; \underset{\text{(side)}^2}{(x + 1)^2} \; \underset{\text{equals}}{=} \; \underset{\text{(hypotenuse)}^2}{(x + 2)^2}$$

$$
\begin{aligned}
x^2 + (x + 1)^2 &= (x + 2)^2 && \text{Pythagorean Theorem} \\
x^2 + x^2 + 2x + 1 &= x^2 + 4x + 4 && \text{Square each term} \\
2x^2 + 2x + 1 &= x^2 + 4x + 4 && \text{Combine like terms} \\
x^2 - 2x - 3 &= 0 && \text{Write in standard form} \\
(x - 3)(x + 1) &= 0 && \text{Factor the left member} \\
x - 3 = 0 \quad \text{or} \quad x + 1 &= 0 && \text{Set each factor equal to 0} \\
x = 3 \qquad\qquad x &= -1
\end{aligned}
$$

Reject -1 as an answer since we are finding the length of a side. Then

$x = 3$, $x + 1 = 4$, and $x + 2 = 5$

and the lengths of the sides of the triangle are 3, 4, and 5 units.

3. The sum of the squares of two consecutive even integers is 52. Find the integers.

Let x = the first even integer, then $x + 2$ = the next consecutive even integer.

$$\underset{\text{(integer)}^2}{x^2} \; \underset{\text{sum}}{+} \; \underset{\text{(integer)}^2}{(x + 2)^2} \; \underset{\text{is}}{=} \; \underset{52}{52}$$

$$
\begin{aligned}
x^2 + (x + 2)^2 &= 52 && \text{Sum of the squares equals 52} \\
x^2 + x^2 + 4x + 4 &= 52 && \\
2x^2 + 4x + 4 &= 52 && \\
2x^2 + 4x - 48 &= 0 && \text{Write in standard form} \\
2(x^2 + 2x - 24) &= 0 && \text{Factor the left member} \\
2(x + 6)(x - 4) &= 0 && \\
x + 6 = 0 \quad \text{or} \quad x - 4 &= 0 && \text{Set each factor equal to zero (Note: } 2 \ne 0) \\
x = -6 \qquad\qquad x &= 4
\end{aligned}
$$

When $x = -6$, then $x + 2 = -4$. When $x = 4$, then $x + 2 = 6$. The integers are -6 and -4 or 4 and 6.

▶ **Quick check**   The sum of the squares of two consecutive *odd* integers is 130. Find the integers.

___ *Mastery points* ___

Can you
■ Solve a quadratic equation by factoring?
■ Solve a quadratic equation by extracting the roots?
■ Solve word problems whose mathematical language yields quadratic equations?

Exercise 6-1

Find the solution set of the following quadratic equations by factoring. See example 6–1 A.

Example $3x^2 - 14x = 0$

Solution

$$x(3x - 14) = 0 \qquad \text{Factor the left member}$$
$$x = 0 \quad \text{or} \quad 3x - 14 = 0 \qquad \text{Set each factor equal to 0}$$
$$x = 0 \qquad\qquad 3x = 14 \qquad \text{Solve for } x$$
$$x = 0 \qquad\qquad x = \frac{14}{3}$$

The solution set is $\left\{0, \dfrac{14}{3}\right\}$.

1. $(x - 3)(x + 4) = 0$ 2. $(x - 7)(x + 1) = 0$ 3. $(3y - 1)(2y + 5) = 0$ 4. $(2z - 3)(3z - 4) = 0$
5. $x^2 - 5x + 6 = 0$ 6. $x^2 - 7x - 8 = 0$ 7. $y^2 - 10y + 25 = 0$ 8. $y^2 - 18y + 81 = 0$
9. $p^2 - 5p = 24$ 10. $z^2 - 12 = 4z$ 11. $m^2 - m = 0$ 12. $q^2 + 3q = 0$
13. $-3y^2 + 27 = 0$ 14. $-9z^2 + 144 = 0$ 15. $2x^2 - 3x - 2 = 0$ 16. $2n^2 + 11n - 6 = 0$
17. $4y^2 + 5y = 6$ 18. $3x^2 - 8 = 10x$ 19. $x - 1 - \dfrac{x^2}{4} = 0$ 20. $\dfrac{x^2}{6} - \dfrac{x}{3} - \dfrac{1}{2} = 0$
21. $\dfrac{x}{2} + \dfrac{7}{2} = \dfrac{4}{x}$ 22. $x + \dfrac{1}{3} = \dfrac{4}{3x}$ 23. $(y + 6)(y - 2) = -7$ 24. $(p + 4)(p - 6) = -16$
25. $(3m + 2)(m - 1) = 4m$ 26. $(3x + 2)(x - 1) = -(7x - 7)$ 27. $3x(x - 3) = (x - 5)(x - 3)$
28. $(y - 1)(y + 4) = 2y(y + 4)$ 29. $(x - 1)^2 = (2x + 5)(x - 1)$

Find the solution set of the following equations by extracting the roots. Express radicals in simplest form. See example 6–1 B.

Example $(4z + 5)^2 = 11$

Solution
$$4z + 5 = \sqrt{11} \qquad \text{or} \qquad 4z + 5 = -\sqrt{11} \qquad \text{Extract the roots}$$
$$4z = -5 + \sqrt{11} \quad \text{or} \qquad 4z = -5 - \sqrt{11} \qquad \text{Add } -5 \text{ to each member.}$$
$$z = \frac{-5 + \sqrt{11}}{4} \quad \text{or} \qquad z = \frac{-5 - \sqrt{11}}{4} \qquad \text{Divide by 4}$$

The solution set is $\left\{\dfrac{-5 + \sqrt{11}}{4}, \dfrac{-5 - \sqrt{11}}{4}\right\}$.

30. $x^2 = 81$ 31. $x^2 = 121$ 32. $3y^2 = 27$ 33. $5z^2 = 245$
34. $p^2 = 20$ 35. $q^2 = 32$ 36. $m^2 - 40 = 0$ 37. $y^2 - 72 = 0$
38. $16p^2 - 400 = 0$ 39. $7y^2 - 56 = 0$ 40. $9x^2 - 162 = 0$ 41. $2n^2 - 100 = 0$
42. $(x + 3)^2 = 16$ 43. $(y + 7)^2 = 36$ 44. $(x - 9)^2 = -144$ 45. $(x - 12)^2 = -121$
46. $(x - 6)^2 = 12$ 47. $(y + 10)^2 = 48$ 48. $(3x - 2)^2 = 25$ 49. $(4x + 1)^2 = 81$
50. $(9y + 1)^2 = -24$ 51. $(10p - 3)^2 = -84$ 52. $(x - 7)^2 = a^2, a > 0$ 53. $(y + 8)^2 = b^2, b > 0$

Solve the following equations for x.

Example $x^2 - 3ax - 4a^2 = 0$

Solution Factoring the left member, we have $(x - 4a)(x + a) = 0$. This is true when

$$x - 4a = 0 \text{ or } x + a = 0$$

Solving each equation for x, we obtain

$$x = 4a \text{ or } x = -a$$

54. $x^2 + 4ax + 3a^2 = 0$

55. $x^2 - 10bx - 24b^2 = 0$

56. $3x^2 - 13xy + 4y^2 = 0$

57. $4x^2 - ax - 14a^2 = 0$

58. $12x^2 = 8ax + 15a^2$

59. $12xy - 6y^2 = 6x^2$

60. $5x^2 - 6y^2 = 7xy$

61. $6x^2 + 5xy = 4y^2$

Solve the following word problems. See example 6-1 A-5 and 6-1 C.

Example The sum of the squares of two consecutive odd integers is 130. Find the integers.

Solution Let $x =$ the first odd integer, then $x + 2 =$ the next consecutive odd integer.

(integer)²	sum	(integer)²	is	130
x^2	$+$	$(x + 2)^2$	$=$	130

$x^2 + (x + 2)^2 = 130$ Sum of the squares equals 130

$x^2 + x^2 + 4x + 4 = 130$ Multiply in left member

$2x^2 + 4x + 4 = 130$ Combine like terms

$2x^2 + 4x - 126 = 0$ Subtract 130 from each member

$2(x^2 + 2x - 63) = 0$ Factor left member

$2(x + 9)(x - 7) = 0$ Factor the trinomial

$x + 9 = 0$ or $x - 7 = 0$ Set each containing the variable factor equal to 0

$x = -9$ $x = 7$ Solve each equation

When $x = -9$, $x + 2 = -7$ and when $x = 7$, $x + 2 = 9$.
The consecutive odd integers are -9 and -7 or 7 and 9.

62. Given $P = 100I - 5I^2$, find I in amperes, $I > 0$, when (a) $P = 420$, (b) $P = 0$.

63. A ball rolls down a slope and travels a distance d defined by the equation $d = 6t + \dfrac{t^2}{2}$ feet in t seconds. How long does it take the ball to roll (a) $d = 32$ feet, (b) $d = 14$ feet?

64. The height h that an object will reach in t seconds if it is propelled vertically upward with an initial velocity V_0 feet per second is defined by the equation

$$h = -16t^2 + V_0t$$

When will the object hit the ground?

65. An object with an initial velocity V_0 accelerates at rate a in time t. The displacement of the object for this time is given by the equation

$$s = V_0t + \frac{1}{2}at^2$$

If $V_0 = 3$ and $a = 6$, when will the displacement s be 6 feet?

66. The sum S of the first n even positive integers is given by $S = n(n + 1)$. Find n when $S = 30$.

67. The sum S of the first n of the numbers 5, 8, 11, . . . , $3n + 2$ is given by $S = \dfrac{1}{2}n(3n + 7)$.

Find n when $S = 98$.

68. If the diagonal of a square is 32 feet, find the length of each side of the square.

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69. If the diagonal of a square is $7\sqrt{2}$ meters long, find the length of each side of the square.

70. The three sides of a right triangle are three consecutive even integers. Find the lengths of the three sides.

71. One leg of a right triangle is 7 inches longer than the other leg. If the hypotenuse is 9 inches longer than the shortest leg, find the lengths of the three sides of the triangle.

72. The longest side of a right triangle is 1 yard longer than twice the length of the shortest side. If the third side measures 15 yards, find the lengths of the other two sides.

73. The square of the sum of two consecutive even integers is 100. Find the integers.

74. The square of the sum of two consecutive odd integers is 144. Find the integers.

75. The sum of the squares of two consecutive odd integers is 130. Find the integers.

76. One integer is 1 more than twice the other integer. Their product is 105. Find the integers.

Review exercises

Simplify the following expressions. Assume all variables are nonzero. Answer with all exponents positive. See section 3–3.

1. $\dfrac{x^{-2}y^3}{x^2y^{-1}}$

2. $(4x^{-3}y^2)^{-2}$

Find the following products. See section 3–2.

3. $(2x - 3)^2$

4. $(x + 7)^2$

Find the solution set of the following equations and inequalities. See sections 2–5 and 6–1.

5. $-3 \le 2x - 1 < 5$

6. $4x^2 - 13x = -3$

6–2 ■ Solution by completing the square

Completing the square

Finding the solution set of quadratic equations by factoring and by extracting the roots required special types of the quadratic equation. Now we develop a method that can be applied to any quadratic equation. The method, called **completing the square,** involves transforming the standard quadratic equation

$$ax^2 + bx + c = 0, a > 0$$

into the form

$$(x + k)^2 = d$$

where k and d are constants. This latter equation can then be solved by extracting the roots as we did in section 6–1.

Consider the identities

$$(x + 8)^2 = x^2 + 16x + 64$$
$$(x - 7)^2 = x^2 - 14x + 49$$

Observe first that the coefficient of x^2 in each case is 1. This is necessary for what we do next. We then consider the relationship between the second term (the

linear term) and the third term (the constant term) of the trinomial. Notice that the constant term in each case is the *square of one-half of the coefficient of* the middle (linear) term, x.

    1. In $x^2 + 16x + 64$, the constant term, 64, is the square of one-half of the coefficient of the middle (linear) term, 16.

$$\left[\frac{1}{2}(16)\right]^2 = 8^2 = 64$$

    2. In $x^2 - 14x + 49$, the constant term, 49, is the square of one-half of the coefficient of the middle (linear) term, -14.

$$\left[\frac{1}{2}(-14)\right]^2 = (-7)^2 = 49$$

Further, we see that the constant term in the binomial square is the number that is one-half of the coefficient of the middle (linear) term of the perfect square trinomial.

    1. Given $x^2 + 16x + 64 = (x + 8)^2$

$$\left[\frac{1}{2}(16)\right] = 8$$

    2. Given $x^2 - 14x + 49 = [x + (-7)]^2 = (x - 7)^2$

$$\left[\frac{1}{2}(-14)\right] = -7$$

We now use these observations to "build" perfect square trinomials by *completing the square* and thereby obtain their equivalent perfect squares.

■ Example 6–2 A

Determine what number must be added to each expression to make it a perfect square trinomial. State the equivalent square of a binomial.

1. $x^2 + 10x$

The coefficient of the linear term, $10x$, is 10. Now we square one-half of 10 to obtain

$$\left[\frac{1}{2}(10)\right]^2 = 5^2 = 25$$

Adding 25 to the given expression, we have

$$x^2 + 10x + 25 = (x + 5)^2$$

2. $x^2 - 7x$

The coefficient of the linear term is -7. The square of one-half of -7 is

$$\left[\frac{1}{2}(-7)\right]^2 = \left(-\frac{7}{2}\right)^2 = \frac{49}{4}$$

Then

$$x^2 - 7x + \frac{49}{4} = \left(x - \frac{7}{2}\right)^2$$

▶ *Quick check* What must be added to $z^2 - 9z$ to make a perfect square trinomial?

Solving by completing the square

We now use this procedure to obtain the solution set of a quadratic equation by completing the square.

■ **Example 6–2 B**

Find the solution set by completing the square.

1. $x^2 - 6x + 5 = 0$

   Isolate $x^2 - 6x$ by subtracting 5 from each member.

   $$x^2 - 6x = -5$$

   $$x^2 - 6x + \left[\frac{1}{2}(-6)\right]^2 = -5 + \left[\frac{1}{2}(-6)\right]^2 \qquad \text{Complete the square}$$

   $$x^2 - 6x + 9 = -5 + 9 \qquad \left[\frac{1}{2}(-6)\right]^2 = 9$$

   $$(x - 3)^2 = 4 \qquad \text{Factor the left member, combine in the right member}$$

   $$x - 3 = 2 \quad \text{or} \quad x - 3 = -2 \qquad \text{Extract the roots}$$
   $$x = 5 \qquad\qquad x = 1$$

   The solution set is $\{1,5\}$.

   Note A common error when solving by this method is *to **fail** to add the same number to each member* when completing the square. Failure to do so changes the equation.

2. $x^2 + 8x - 2 = 0$

   $$x^2 + 8x = 2 \qquad \text{Isolate variable terms}$$

   $$x^2 + 8x + \left[\frac{1}{2}(8)\right]^2 = 2 + \left[\frac{1}{2}(8)\right]^2 \qquad \text{Complete the square}$$

   $$x^2 + 8x + 16 = 2 + 16 \qquad \left[\frac{1}{2}(8)\right]^2 = 16$$

   $$(x + 4)^2 = 18 \qquad \text{Factor the left member and add in the right member}$$

   $$x + 4 = \sqrt{18} \quad \text{or} \quad x + 4 = -\sqrt{18} \qquad \text{Extract the roots}$$
   $$x + 4 = 3\sqrt{2} \qquad\quad x + 4 = -3\sqrt{2} \qquad \sqrt{18} = 3\sqrt{2}$$
   $$x = -4 + 3\sqrt{2} \qquad x = -4 - 3\sqrt{2} \qquad \text{Solve for } x$$

   The solution set is $\{-4 - 3\sqrt{2}, -4 + 3\sqrt{2}\}$.

   Completing the square can be used *only* when the coefficient of x^2 is 1. When the coefficient is other than 1, we often divide each member of the equation by that coefficient.

3. $$3x^2 - 12x - 9 = 0$$

   $$x^2 - 4x - 3 = 0 \qquad \text{Divide each term by 3 to obtain leading coefficient of 1}$$
   $$x^2 - 4x = 3 \qquad \text{Isolate variable terms}$$

   $$x^2 - 4x + \left[\frac{1}{2} \cdot (-4)\right]^2 = 3 + \left[\frac{1}{2} \cdot (-4)\right]^2 \qquad \text{Complete the square}$$

   $$x^2 - 4x + 4 = 3 + 4$$
   $$(x - 2)^2 = 7 \qquad \text{Factor the left member; combine in the right member}$$
   $$x - 2 = \sqrt{7} \quad \text{or} \quad x - 2 = -\sqrt{7} \qquad \text{Extract the roots}$$
   $$x = 2 + \sqrt{7} \qquad\qquad x = 2 - \sqrt{7}$$

   The solution set is $\{2 - \sqrt{7}, 2 + \sqrt{7}\}$.

▶ *Quick check*   Find the solution set of $3y^2 - 6y - 3 = 0$ by completing the square.

In summary, to find the solution set of the quadratic equation $ax^2 + bx + c = 0$, $a > 0$, by completing the square, we follow this procedure.

To solve quadratic equations by completing the square

1. If $a = 1$, proceed to step 2. If $a \neq 1$, divide each term of the equation by a, if necessary to complete the square.
2. Write the equation with the variable terms in the left member and the constant in the right member.
3. Add to each member of the equation the square of one-half of the coefficient of the linear term.
4. Write the left member as a perfect square and combine in the right member.
5. Extract the roots and solve the resulting linear equations.

Mastery points

Can you
- Complete the square of a binomial of the form $x^2 + kx$?
- Find the solution set of a quadratic equation by completing the square?

Exercise 6–2

Determine what number must be added to each expression to make a perfect square trinomial. State the equivalent binomial square. See example 6–2 A.

Example $z^2 - 9z$

Solution $\left[\frac{1}{2}(-9)\right]^2 = \left(-\frac{9}{2}\right)^2 = \frac{81}{4}$ Square one-half of the coefficient of z

$z^2 - 9z + \frac{81}{4} = \left(z - \frac{9}{2}\right)^2$ Add $\frac{81}{4}$ to the given expression and factor

1. $x^2 + 4x +$?
2. $x^2 + 8x +$?
3. $y^2 - 18y +$?
4. $z^2 - 24z +$?
5. $p^2 + 2p +$?
6. $m^2 - 30m +$?
7. $x^2 + 3x +$?
8. $y^2 + y +$?
9. $w^2 - 11w +$?
10. $q^2 - 5q +$?
11. $x^2 + 13x +$?
12. $y^2 - 15y +$?

Find the solution set by completing the square. See example 6–2 B.

Example $3y^2 - 6y - 3 = 0$

Solution

$$y^2 - 2y - 1 = 0 \qquad \text{Divide each term by 3}$$
$$y^2 - 2y = 1 \qquad \text{Isolate variable terms}$$
$$y^2 - 2y + \left[\frac{1}{2} \cdot (-2)\right]^2 = 1 + \left[\frac{1}{2} \cdot (-2)\right]^2 \qquad \text{Complete the square}$$
$$y^2 - 2y + 1 = 1 + 1 \qquad \left[\frac{1}{2} \cdot (-2)\right]^2 = 1$$
$$(y - 1)^2 = 2 \qquad \text{Factor left member and add in right member}$$
$$y - 1 = \sqrt{2} \quad \text{or} \quad y - 1 = -\sqrt{2} \qquad \text{Extract the roots}$$
$$y = 1 + \sqrt{2} \qquad y = 1 - \sqrt{2} \qquad \text{Solve for } y$$

The solution set is $\{1 - \sqrt{2}, 1 + \sqrt{2}\}$.

13. $x^2 + 12x + 11 = 0$ 14. $x^2 + 5x + 4 = 0$ 15. $y^2 - 11y + 10 = 0$ 16. $p^2 - 4p = 8$

17. $n^2 + 8n = -25$ 18. $x^2 + 6x = -10$ 19. $x^2 - 8x = 0$ 20. $x^2 + 4x = 0$

21. $y^2 = 3 - y$ 22. $x^2 + 2 = -4x$ 23. $-2x^2 + 4 = -6x$ 24. $2n = 4 - n^2$

25. $2x^2 + 3x - 2 = 0$ 26. $2y^2 - 4y - 3 = 0$ 27. $3x^2 - 12x + 3 = 0$ 28. $1 - z^2 = 3z$

29. $4u^2 - 4u = 3$ 30. $4x^2 + 12x + 4 = 0$ 31. $5m^2 - 5m + 1 = 0$ 32. $5q^2 + 4q + 1 = 0$

33. $(x + 2)(x - 3) = 1$ 34. $(2y - 1)(y + 5) = -3$ 35. $(3x + 1)^2 = (x - 2)^2$ 36. $(2y - 3)^2 = (y + 4)^2$

37. $p(2p - 1) = p + 1$ 38. $8p(p + 3) = 2p - 5$ 39. $\frac{1}{2}x^2 - \frac{3}{4}x = 1$ 40. $y^2 - \frac{1}{3}y = \frac{2}{3}$

41. $\frac{1}{2}x^2 - \frac{2}{3}x - 1 = 0$ 42. $z^2 + \frac{1}{2}z - \frac{3}{4} = 0$ 43. $x^2 - \frac{3}{2} = 2x$ 44. $\frac{1}{5}n^2 = 1 - 2n$

45. $\frac{5}{x} - 2x + 3 = 0$ 46. $\frac{3}{x} - 2 = 2x$ 47. $5 - \frac{2}{t} = \frac{3}{t^2}$ 48. $3 + \frac{5}{p} = \frac{1}{p^2}$

Solve the following word problems by completing the square.

49. When an object is thrown downward with an initial velocity of 9 feet per second, the relationship between the distance s it travels in time t is given by

$$s = 9t + 16t^2$$

How long does it take for the object to fall 100 feet?

50. When an object is dropped from rest, the distance s that the object falls is given by the equation

$$s = 16t^2$$

How long does it take the object to hit the bottom of a gorge when it is dropped a distance of 205 feet from a bridge across the gorge?

51. The supply equation for a specific product is given by

$$S = 32p + p^2$$

where p cents is the price per unit of the product. What should the price to the nearest cent be when the supply S is 500 units?

52. The demand equation for a specific product is given by

$$D = 36p + p^2$$

where p is the price per 1,000 units. What should the price to the nearest cent per 1,000 units be when the demand D is 100,000 units?

53. The radius r of a circular arch having height h and span b is given by

$$r = \frac{(b^2 + 4h^2)}{8h}$$

Find h when $b = 10$ and $r = 13$.

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54. The square of a number added to three times the number is 6. Find the number.

55. The square of twice a number less the number is 4. Find the number.

56. The square of the difference between four times a number and 5 is 24. Find the number.

57. The formula for the volume of a cylinder with height h and radius r is

$$V = \pi r^2 h$$

Find r when V is 235 cubic meters and $h = 10$ meters.

Review exercises

Perform the indicated multiplications. See section 5–6.

1. $\sqrt{3}(2 - \sqrt{5})$ **2.** $(2 - \sqrt{3})(2 + \sqrt{3})$ **3.** $(\sqrt{2} + \sqrt{3})(2\sqrt{3} - 3\sqrt{2})$ **4.** $(3\sqrt{2} - 3)^2$

Divide the following expressions. See section 4–5.

5. $\dfrac{16x^3 - 8x^2 + 4x}{4x^2}$

6. $(4x^2 - 3x - 2) \div (x - 3)$

Evaluate $b^2 - 4ac$ given the following values of a, b, and c. See section 1–5.

7. $a = 2, b = -3, c = 1$ **8.** $a = 3, b = 5, c = -3$ **9.** $a = 1, b = 2, c = 4$

6–3 ■ Solution by quadratic formula

The quadratic formula

In the previous sections, we have found the solution set of quadratic equations by factoring, extracting the roots, and completing the square. Even though the solution set of any quadratic equation can be found by completing the square, this can be a time-consuming chore. In this section, we will use the method of completing the square to develop a general formula that will always find the solution set. We call this formula the **quadratic formula.**

Given the quadratic equation in standard form,

$$ax^2 + bx + c = 0 \text{ (assume } a > 0)$$

where a, b, and c are real numbers, we can solve for x by completing the square.

$$x^2 + \frac{b}{a}x + \frac{c}{a} = 0 \qquad \text{Divide each term by the coefficient } a \text{ of } x^2$$

$$x^2 + \frac{b}{a}x = -\frac{c}{a} \qquad \text{Subtract } \frac{c}{a} \text{ from each member}$$

$$\left[\frac{1}{2}\left(\frac{b}{a}\right)\right]^2 = \left(\frac{b}{2a}\right)^2 = \frac{b^2}{4a^2} \qquad \text{Square one-half of the coefficient of } x$$

$$x^2 + \frac{b}{a}x + \frac{b^2}{4a^2} = \left(-\frac{c}{a}\right) + \left(\frac{b^2}{4a^2}\right) \qquad \text{Add } \left(\frac{1}{2} \cdot \frac{b}{a}\right)^2 = \frac{b^2}{4a^2} \text{ to each member}$$

$$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a^2} \qquad \text{Factor and subtract}$$

$$x + \frac{b}{2a} = \pm \sqrt{\frac{b^2 - 4ac}{4a^2}} \qquad \text{Extract the roots}$$

$$x = -\frac{b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a} \qquad \text{Add } -\frac{b}{2a} \text{ to each member}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \qquad \text{Add and subtract over the common denominator}$$

The solution set is

$$\left\{\frac{-b + \sqrt{b^2 - 4ac}}{2a}, \frac{-b - \sqrt{b^2 - 4ac}}{2a}\right\} \text{ or } \left\{\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}\right\}.$$

The quadratic formula

If $ax^2 + bx + c = 0$, then

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}, a > 0$$

where a is the coefficient of the second-degree term, b of the first-degree term, and c is the constant.

Solving by quadratic formula

To find the solution set of any quadratic equation by using the quadratic formula, we only need to substitute the numerical values for a, b, and c into the formula and to simplify the result. Consider the equation

$$x^2 + 4x - 12 = 0$$

Since $a = 1$, $b = 4$, and $c = -12$, we substitute in the quadratic formula to obtain

$$x = \frac{-4 \pm \sqrt{(4)^2 - 4(1)(-12)}}{2(1)}$$
$$= \frac{-4 \pm \sqrt{16 + 48}}{2}$$
$$= \frac{-4 \pm \sqrt{64}}{2}$$
$$= \frac{-4 \pm 8}{2}$$

$$x = \frac{-4 + 8}{2} = \frac{4}{2} = 2 \quad \text{or} \quad x = \frac{-4 - 8}{2} = \frac{-12}{2} = -6$$

The solution set is $\{-6, 2\}$.

We now summarize the procedure for solving a quadratic equation by using the quadratic formula.

To solve quadratic equations using the quadratic formula

1. Write the equation in standard form (if necessary).
2. Identify the numerical values of a, b, and c.
3. Substitute these values into the quadratic formula.
4. Simplify the resulting expression.

■ Example 6–3 A

Find the solution set using the quadratic formula.

1. $x^2 = 5 - 3x$

   Write the equation in standard form.

   $x^2 + 3x - 5 = 0$ Add $3x - 5$ to each member

   $a = 1, b = 3, c = -5.$ Identify a, b, and c

$$x = \frac{-(3) \pm \sqrt{(3)^2 - 4(1)(-5)}}{2(1)}$$

Replace a with 1, b with 3, and c with −5

$$x = \frac{-3 \pm \sqrt{9 + 20}}{2}$$

Perform indicated operations

$$x = \frac{-3 \pm \sqrt{29}}{2}$$

Simplify

The solution set is $\left\{ \dfrac{-3 + \sqrt{29}}{2}, \dfrac{-3 - \sqrt{29}}{2} \right\}$.

2. $2y^2 - 4y + 1 = 0$

$a = 2, b = -4, c = 1.$ Identify a, b, and c

$$y = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(2)(1)}}{2(2)}$$

Replace a with 2, b with −4, and c with 1

$$y = \frac{4 \pm \sqrt{16 - 8}}{4}$$

Perform indicated operations

$$y = \frac{4 \pm \sqrt{8}}{4}$$

Simplify

$$y = \frac{4 \pm 2\sqrt{2}}{4}$$

$\sqrt{8} = \sqrt{4 \cdot 2} = 2\sqrt{2}$

$$y = \frac{2(2 \pm \sqrt{2})}{4}$$

Factor 2 in the numerator

$$y = \frac{2 \pm \sqrt{2}}{2}$$

Reduce by 2

The solution set is $\left\{ \dfrac{2 - \sqrt{2}}{2}, \dfrac{2 + \sqrt{2}}{2} \right\}$.

3. $4 - \dfrac{3}{x} + \dfrac{4}{x^2} = 0 \quad (x \neq 0)$

Multiply each term by the LCM of x and x^2, which is x^2.

$$x^2 \cdot 4 - x^2 \cdot \frac{3}{x} + x^2 \cdot \frac{4}{x^2} = x^2 \cdot 0$$

Multiply each term by x^2

$$4x^2 - 3x + 4 = 0$$

Clear the denominators

$a = 4, b = -3,$ and $c = 4.$ Identify a, b, and c

$$x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(4)(4)}}{2(4)}$$

Replace a with 4, b with −3, and c with 4

$$= \frac{3 \pm \sqrt{9 - 64}}{8}$$

$$= \frac{3 \pm \sqrt{-55}}{8}$$

$$= \frac{3 \pm i\sqrt{55}}{8}$$

$\sqrt{-55} = i\sqrt{55}$

The solution set is $\left\{ \dfrac{3 + i\sqrt{55}}{8}, \dfrac{3 - i\sqrt{55}}{8} \right\}$.

▶ *Quick check* Find the solution set of $3y^2 - 6y + 2 = 0$ using the quadratic formula.

To solve any quadratic equation, we use the following steps.

Solving a quadratic equation

1. Write the equation in standard form. Clear fractions if necessary.
2. Check to see if the polynomial expression factors. If so, solve by factoring.
3. If $b = 0$, solve by extracting the roots.
4. Use the quadratic formula.

The discriminant

The general quadratic equation $ax^2 + bx + c = 0$ has two solutions x_1 and x_2 such that, using the quadratic formula,

$$x_1 = \frac{-b + \sqrt{b^2 - 4ac}}{2a} \quad \text{and} \quad x_2 = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

We can determine the nature of the solutions x_1 and x_2 (that is, rational, irrational, or complex) by using the radicand, $b^2 - 4ac$, where a, b, and c are **rational** numbers. For this reason, we call $b^2 - 4ac$ the **discriminant** of the quadratic equation $ax^2 + bx + c = 0$. The nature of the solutions can be determined as follows:

Nature of the solutions of a quadratic equation

When the discriminant $b^2 - 4ac$ is

Zero	Positive	Negative
$b^2 - 4ac = 0$	$b^2 - 4ac > 0$	$b^2 - 4ac < 0$
One *rational* solution of multiplicity two, namely $-\dfrac{b}{2a}$	If $b^2 - 4ac$ is a. a perfect square, two distinct *rational* solutions b. not a perfect square, *two* distinct *irrational* solutions	Two distinct *complex* (nonreal) solutions

Example 6–3 B

Use the discriminant to decide the number and the nature of the solutions of the given quadratic equation.

1. $4x^2 + 12x + 9 = 0$

   Since $a = 4$, $b = 12$, and $c = 9$, then

   $$b^2 - 4ac = (12)^2 - 4(4)(9) = 144 - 144 = 0$$

   There is only *one rational* solution of multiplicity two and that solution is

   $$\frac{-b}{2a} = \frac{-12}{2(4)} = \frac{-3}{2} = -\frac{3}{2}$$

2. $3x^2 - 4 = 4x$

Write the equation in standard form, $3x^2 - 4x - 4 = 0$. Thus $a = 3$, $b = -4$, and $c = -4$. Then

$$b^2 - 4ac = (-4)^2 - 4(3)(-4) = 16 + 48 = 64$$

Since $64 = 8^2$, the discriminant is positive, a perfect square, and there are *two distinct rational* solutions.

Note The discriminant can be used to determine if the trinomial is factorable. In examples 1 and 2, where there is either one rational solution or two rational solutions, the equation factors,

$$4x^2 + 12x + 9 = (2x + 3)^2 \quad \text{and}$$
$$3x^2 - 4x - 4 = (3x + 2)(x - 2)$$

and the discriminant is a perfect square.

3. $x^2 = 4x + 6$

We must first write the equation in standard form, $x^2 - 4x - 6 = 0$. Then $a = 1$, $b = -4$, and $c = -6$, and

$$b^2 - 4ac = (-4)^2 - 4(1)(-6) = 16 + 24 = 40$$

Since 40 is not a perfect square, but is positive, there are *two distinct irrational* solutions, and the polynomial $x^2 - 4x - 6$ does not factor.

4. $2y^2 - 3y + 5 = 0$

Since $a = 2$, $b = -3$, and $c = 5$, then

$$b^2 - 4ac = (-3)^2 - 4(2)(5) = 9 - 40 = -31$$

The discriminant is negative, the equation has *two distinct complex* (nonreal) solutions, and the polynomial does not factor.

▶ *Quick check* Decide the number and the nature of the solutions of $3x^2 + 2x - 3 = 0$ using the discriminant. ■

Mastery points

Can you
- Identify the numerical values of a, b, and c in a standard quadratic equation?
- Solve a quadratic equation by using the quadratic formula?
- Use the discriminant to determine the nature of the solutions?

Exercise 6–3

Find the solution set of each quadratic equation using the quadratic formula in exercises 1–10. See example 6–3 A. In exercises 11–37, use any convenient method.

Example $3y^2 - 6y + 2 = 0$

Solution $y = \dfrac{-(-6) \pm \sqrt{(-6)^2 - 4(3)(2)}}{2(3)}$ Replace a with 3, b with −6, and c with 2

$= \dfrac{6 \pm \sqrt{36 - 24}}{6}$ Multiply in radicand

$= \dfrac{6 \pm \sqrt{12}}{6}$ Subtract in radicand

$= \dfrac{6 \pm 2\sqrt{3}}{6}$ $\sqrt{12} = 2\sqrt{3}$

$= \dfrac{2(3 \pm \sqrt{3})}{2 \cdot 3}$ Factor the common factor of 2 from the numerator and the denominator

$= \dfrac{3 \pm \sqrt{3}}{3}$ Reduce to lowest terms

The solution set is $\left\{ \dfrac{3 + \sqrt{3}}{3}, \dfrac{3 - \sqrt{3}}{3} \right\}$.

1. $p^2 = -5p - 7$
2. $y^2 + 6 = 2y$
3. $3x - 5 = x^2$
4. $18 = 10x - x^2$
5. $2x^2 - 7x + 6 = 0$
6. $3y^2 - 5y - 6 = 0$
7. $4z^2 - 8z + 1 = 0$
8. $9p^2 - 8p + 7 = 0$
9. $3q^2 - 2q + 7 = 0$
10. $3z^2 - 4z = -3$
11. $y^2 + 6y - 16 = 0$
12. $z^2 + 6z + 9 = 0$
13. $p^2 - 14p + 49 = 0$
14. $x^2 - 28 = 0$
15. $3y^2 = 20$
16. $4z^2 - 3z = 0$
17. $2x = 5x^2$
18. $m^2 - 2m = 4$
19. $9x^2 - 12x + 4 = 0$
20. $4y^2 + 20y + 25 = 0$
21. $9y^2 - 12y = -5$
22. $2t^2 = 6t - 5$
23. $11 - 6m = 9m^2$
24. $2v = 5v^2 - 2$
25. $x = 2x^2 + 7$
26. $3x - \dfrac{2}{x} + 5 = 0$
27. $2y^2 - \dfrac{7}{2} = \dfrac{y}{2}$
28. $\dfrac{2}{3}x^2 - \dfrac{1}{3} = x$
29. $\dfrac{2}{3}y - \dfrac{1}{3} = \dfrac{4}{9}y^2$
30. $\dfrac{2p}{3} - \dfrac{p^2}{4} = 1$
31. $\dfrac{3}{4}q^2 = \dfrac{1}{2}q + 4$
32. $\dfrac{1}{x + 2} + \dfrac{1}{x - 3} - 2 = 0$
33. $\dfrac{3}{y - 5} - \dfrac{2}{y + 1} + 3 = 0$
34. $\dfrac{1}{2} - \dfrac{3}{2x + 3} = \dfrac{3}{x - 4}$
35. $(z - 3)(z + 2) = 2z - 3$
36. $(x + 6)(x - 5) = 10 - x$
37. $(2x + 1)^2 = (x - 3)^2$

Solve the following equations for x in terms of the other variables or constants. Assume that all other variables are positive real numbers.

Example $2x^2 - 3xy - 5y^2 = 0$

Solution Here $a = 2$, $b = -3y$ (coefficient of x), and $c = -5y^2$ (term not containing x). Then

$x = \dfrac{-(-3y) \pm \sqrt{(-3y)^2 - 4(2)(-5y^2)}}{2(2)}$ Replace a with 2, b with −3y, and c with −5y²

$= \dfrac{3y \pm \sqrt{9y^2 + 40y^2}}{4}$ Simplify

$$= \frac{3y \pm \sqrt{49y^2}}{4}$$

$$= \frac{3y \pm 7y}{4}$$

Thus

$$x = \frac{3y + 7y}{4} = \frac{10y}{4} = \frac{5}{2}y \text{ or } x = \frac{3y - 7y}{4} = \frac{-4y}{4} = -y$$

Therefore $x = \frac{5}{2}y$ or $x = -y$.

38. $x^2 - xy - 2y^2 = 0$ **39.** $x^2 - 3xy - 18y^2 = 0$ **40.** $2x^2 - 3ax + 5a^2 = 0$

41. $4x^2 + 2x - 3y = 0$ **42.** $2x^2 - 3x + 4a = 0$ **43.** $x^2 - 4ax + 3a = 0$

Using the discriminant $b^2 - 4ac$, determine the number and the nature of the solutions of each quadratic equation. See example 6-3 B.

Example $3x^2 + 2x - 3 = 0$

Solution $b^2 - 4ac = (2)^2 - 4(3)(-3)$ Replace a with 3, b with 2, and c with −3
$$= 4 + 36$$ Multiply as indicated
$$= 40$$

Since 40 is positive and not a perfect square, then the equation has *two distinct irrational* solutions.

44. $x^2 + 4x + 1 = 0$ **45.** $x^2 - 3x - 5 = 0$ **46.** $2y^2 - y + 1 = 0$

47. $3x^2 + 3x + 4 = 0$ **48.** $4y^2 - 4y + 1 = 0$ **49.** $9x^2 - 30x + 25 = 0$

50. $4y^2 - y = 10$ **51.** $3m^2 - 5m = 2$ **52.** $y^2 + 6 = -2y$

53. $x^2 = -4 + 6x$ **54.** $y^2 = 20$ **55.** $m^2 - 18 = 4m$

56. $5t^2 - 3t = 0$ **57.** $7p^2 = 8p + 2$ **58.** $x^2 - \frac{1}{2}x + \frac{3}{5} = 0$

59. $x^2 - 4x + \frac{9}{4} = 0$ **60.** $\frac{y^2}{2} + 5y = 1$ **61.** $\frac{m^2}{4} + \frac{3m}{2} = 5$

Solve the following using the quadratic formula.

62. The distance s through which an object will fall in t seconds is given by

$$s = \frac{1}{2}gt^2$$

where $g = 32$ feet per second per second (32 ft/sec²). Find t to the nearest tenth of a second when (a) $s = 96$ feet, (b) $s = 60$ feet.

63. A metal strip is shaped into a right triangle as shown in the diagram. If $a = x$, $b = x + 3$, and $c = x + 6$, find x.
(*Hint:* Use the Pythagorean Theorem previously discussed.)

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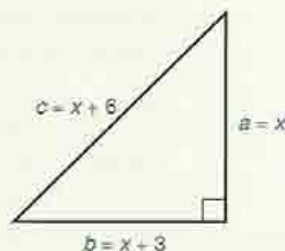
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64. In a given electric circuit, the relationship between I (in amperes), E (in volts), and R (in ohms) is given by

$$I^2R + IE = 8{,}000$$

Find I ($I > 0$) when $R = 4$ and $E = 100$.

65. Using the formula

$$s = v_0t + \frac{1}{2}at^2$$

find t when (a) $s = 8$, $v_0 = 3$, $a = 4$;
(b) $s = 80$, $v_0 = 36$, $a = 32$. ($t > 0$)

66. In chemistry, an equation in connection with equilibrium in liquid flow is given by

$$k = \frac{x^2}{(a - x)(b - x)}$$

Solve for x if $a = 1$, $b = 2$, and $k = 3$.

67. If P dollars are invested at $r\%$ interest compounded annually, after two years the worth A in dollars is given by

$$A = P(1 + 0.01r)^2$$

If the amount $A = \$1{,}200$ after two years when $P = \$1{,}000$ is invested, find the rate of interest r, to the nearest tenth of a percent.

Review exercises

Simplify the following radical expressions. See sections 5–4, 5–5, and 5–6.

1. $4\sqrt{7} - 5\sqrt{63}$

2. $\sqrt{\dfrac{9}{25}}$

3. $(3 - \sqrt{5})^2$

4. Multiply $(3 - 2i)(4 + 5i)$. See section 5–7.

5. Subtract $(2 - \sqrt{-3}) - (4 + \sqrt{-3})$. See section 5–7.

6. Subtract $(2x^5)^2 - (3x^2)^3x^4$. See section 3–3.

6–4 ■ Applications of quadratic equations

A number of physical situations generate quadratic equations. Because of this, there may be two answers to the problem. Sometimes, because of the nature of the situation, only one of the answers is logical. For example, it would not be feasible to accept -25 as the measurement of a dimension of a room or $\dfrac{7}{6}$ as the number of books on a shelf. These answers are not physically logical and would, therefore, be rejected as solutions to an applied problem.

■ **Example 6–4 A**

1. When a ball is thrown straight upward into the air, the equation

$$s = -16t^2 + 80t + 44$$

gives the distance s in feet that the ball is above the ground t seconds after it is thrown. How long does it take for the ball to hit the ground?

The ball hits the ground when $s = 0$. Thus we have

$$0 = -16t^2 + 80t + 44 \qquad \text{Replace } s \text{ with } 0$$
$$16t^2 - 80t - 44 = 0 \qquad \text{Multiply each member by } -1$$
$$4(4t^2 - 20t - 11) = 0 \qquad \text{Factor in the left member}$$
$$4(2t - 11)(2t + 1) = 0$$
$$2t - 11 = 0 \quad \text{or} \quad 2t + 1 = 0 \qquad \text{Set each variable factor equal to } 0$$
$$t = \frac{11}{2} \qquad\qquad t = -\frac{1}{2}$$

Since t represents time, we reject $t = -\dfrac{1}{2}$, so $t = \dfrac{11}{2}$ or $5\dfrac{1}{2}$. The ball hits

the ground in $5\dfrac{1}{2}$ seconds.

Right triangle problem

2. Find the length of side b of the given right triangle if
c (the hypotenuse) $= 13$ units and side b is 7 units longer than side a.

$c = 13$

a

$b = a + 7$
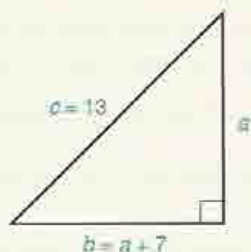
Use the Pythagorean Theorem, $a^2 + b^2 = c^2$. Since b is 7 units longer than a,
$b = a + 7$.

$$a^2 + (a + 7)^2 = (13)^2 \qquad \text{Replace } b \text{ with } a + 7 \text{ and } c \text{ with } 13$$
$$a^2 + a^2 + 14a + 49 = 169$$
$$2a^2 + 14a - 120 = 0$$
$$2(a^2 + 7a - 60) = 0$$
$$2(a + 12)(a - 5) = 0$$
$$a + 12 = 0 \quad \text{or} \quad a - 5 = 0$$
$$\text{Then } a = -12 \text{ or } a = 5.$$

We reject $a = -12$, since the dimensions of a triangle cannot be negative.

$a = 5$ and $b = a + 7 = 5 + 7 = 12$. Side b is 12 units long.

Geometric problem

3. A rectangle has an area, A, of 65 square centimeters. If the length ℓ of the
rectangle is 3 centimeters more than twice the width w, find the dimensions of
the rectangle ($A = \ell w$).

w

$\ell = 2w + 3$
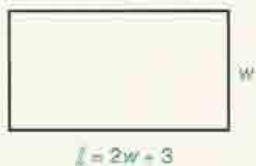
Let $w =$ the width of the rectangle, then $\ell = 2w + 3$.

$$65 = (2w + 3)w \qquad \text{In } A = \ell w, \text{ replace } A \text{ with } 65 \text{ and } \ell \text{ with } 2w + 3$$

$$65 = 2w^2 + 3w$$
$$2w^2 + 3w - 65 = 0$$
$$(2w + 13)(w - 5) = 0$$
$$\text{Then } 2w + 13 = 0 \quad \text{or} \quad w - 5 = 0.$$

Since $w = -\dfrac{13}{2}$ when $2w + 13 = 0$ and $w = 5$ when $w - 5 = 0$, then

$$w = -\dfrac{13}{2} \text{ or } w = 5.$$

Reject $w = -\dfrac{13}{2}$ since we cannot have a negative width. Then $w = 5$ and $\ell = 2w + 3 = 2(5) + 3 = 13$. The rectangle is 5 centimeters wide and 13 centimeters long.

Work problem

4. Two pipes when opened can fill a tank in 5 hours. If one pipe takes 2 hours less to fill the tank than the other one does, how long will it take each pipe to fill the tank alone? (Round off to the nearest tenth.)

Let x = time in hours that the faster pipe takes to fill the tank.
Then $x + 2$ = time in hours that the slower pipe takes to fill the tank.

Faster pipe fills $\dfrac{1}{x}$ of the tank per hour. Slower pipe fills $\dfrac{1}{x + 2}$ of the tank per hour. Together they fill $\dfrac{1}{5}$ of the tank per hour.

faster pipe $\quad+\quad$ slower pipe $\quad=\quad$ together

$$\frac{1}{x} \quad + \quad \frac{1}{x + 2} \quad = \quad \frac{1}{5}$$

$$5x(x + 2) \cdot \frac{1}{x} + 5x(x + 2) \cdot \frac{1}{x + 2} = 5x(x + 2) \cdot \frac{1}{5}$$

> Multiply each term by the LCM $5x(x + 2)$

$$5(x + 2) + 5x = x(x + 2)$$
$$5x + 10 + 5x = x^2 + 2x$$
$$10x + 10 = x^2 + 2x$$
$$x^2 - 8x - 10 = 0$$

> Write in standard form

Since $x^2 - 8x - 10$ is not factorable, we use the quadratic formula.

$$x = \frac{-(-8) \pm \sqrt{(-8)^2 - 4(1)(-10)}}{2(1)}$$

> Replace a with 1, b with -8, and c with -10

$$= \frac{8 \pm \sqrt{64 + 40}}{2}$$
$$= \frac{8 \pm \sqrt{104}}{2}$$
$$= \frac{8 \pm 2\sqrt{26}}{2}$$

> $\sqrt{104} = \sqrt{4 \cdot 26}$
> $= 2\sqrt{26}$

$$= \frac{2(4 \pm \sqrt{26})}{2}$$
$$= 4 \pm \sqrt{26}$$

Since $4 - \sqrt{26}$ is a negative time, we reject that solution. So

$$x = 4 + \sqrt{26} \approx 4 + 5.1 \approx 9.1$$
$$x + 2 = (4 + \sqrt{26}) + 2 \approx 6 + 5.1 \approx 11.1$$

It would take the faster pipe approximately 9.1 hours and the slower pipe approximately 11.1 hours to fill the tank alone.

▶ *Quick check* a. When a ball is thrown straight up into the air, the equation $s = -16t^2 + 80t + 96$ gives the distance s that the ball is above the ground t seconds after it is thrown. How long does it take for the ball to hit the ground?
b. A rectangular plot of ground has an area of 161 square meters. If the length of the rectangle is two more than three times the width, what are the dimensions of the rectangle? ■

--- *Mastery points* ---

Can you

- Substitute and solve physical formulas that are quadratic?
- Solve word problems involving the use of a right triangle and the relationship $a^2 + b^2 = c^2$?
- Solve word problems involving the areas of geometric figures?
- Solve word problems involving economic relationships?
- Solve work problems?

Exercise 6-4

Solve the following using quadratic equations. Compute all answers to the nearest tenth where necessary. See example 6-4 A-1.

Example When a ball is thrown straight up into the air, the equation $s = -16t^2 + 80t + 96$ gives the distance s that the ball is above the ground t seconds after it is thrown. How long does it take for the ball to hit the ground?

Solution

$$0 = -16t^2 + 80t + 96 \qquad \text{Replace } s \text{ with } 0$$
$$16t^2 - 80t - 96 = 0 \qquad \text{Multiply equation by } -1 \text{ and interchange members}$$
$$16(t^2 - 5t - 6) = 0 \qquad \text{Factor the left member}$$
$$16(t - 6)(t + 1) = 0$$
$$t - 6 = 0 \quad \text{or} \quad t + 1 = 0 \qquad \text{Set each factor equal to } 0$$
$$t = 6 \qquad\qquad t = -1$$

Reject $t = -1$ because time cannot be negative. The ball will strike the ground 6 seconds after being thrown.

1. The distance s a body falls when air resistance is neglected is given by

   $$s = v_0 t + 16t^2$$

   where s is in feet, t is in seconds, and v_0 in feet per second is the initial velocity of the body. How long will it take a body to fall 80 feet if it has an initial velocity v_0 of 64 feet per second?

2. Using the information in exercise 1, how long will it take the body in a vacuum to fall 80 feet if the body starts from rest? (*Hint:* $v_0 = 0$ feet per second.)

3. Using the formula in exercise 1, how long will it take a body in a vacuum to fall 240 feet if the initial velocity is $v_0 = 32$ feet per second?

4. An object fired vertically into the air with an initial velocity of v_0 feet per second will be at a distance h in feet, t seconds after launching, determined by the equation

   $$h = v_0 t - 16t^2$$

   If the initial velocity is 96 feet per second, how long will it take the object to reach a height of 80 feet?

5. Using the formula in exercise 4, how long will it take for the object to return to the ground? (*Hint:* $h = 0$ when the object is on the ground.)

6. Using exercise 4, find the time when the object will be 12 feet off the ground.

7. An object is dropped from the top of the Washington Monument (555 feet tall). How long will it take the object to strike the ground? (*Hint:* Use exercise 1.)

8. The current in an electric circuit flows according to the equation

$$I = 18t - 12t^2$$

where I is the current in amperes and t is the time in seconds. In how many seconds will there be 6 amperes of current?

9. Using the formula in exercise 8, in how many seconds t will there be no current?

10. In a given circuit, the relationship between I (in amperes), E (in volts), and R (in ohms) is given by

$$I^2R + IE = 6,000$$

Find I when $E = 60$ volts and $R = 3$ ohms.

11. Using exercise 10, find I when $E = 50$ volts and $R = 5$ ohms.

Business problems

14. The demand equation for a specific commodity is given by

$$D = \frac{2,000}{p}$$

where D is the demand for the specific commodity at price p dollars per unit. If the supply equation is given by

$$S = 300p - 400$$

where S is the quantity of the commodity that the supplier is willing to supply at p dollars per unit, find the equilibrium price.
(*Note:* Equilibrium price occurs when $D = S$.)

15. Suppose that a manufacturer of ballpoint pens finds the demand equation to be

$$D = 24 - p^2$$

and the supply equation to be

$$S = p^2 + 2p$$

where p is the price of each pen in dollars. What is the equilibrium price? (See note in exercise 14.)

16. In exercise 15, at what price, to the nearest cent where necessary, (a) is there no demand, (b) will the quantity that the supplier is willing to sell be zero?

12. The formula for rating engine horsepower, hp, based on an average effective pressure on the piston of 67 pounds per square inch at a piston speed of 1,000 feet per minute is given by

$$hp = 0.4D^2 \times N$$

where D is the diameter of the piston bore and N is the number of pistons. Find D when an 8-cylinder engine has 36 horsepower.

13. A polygon is a closed geometric plane figure that has n sides, where $n \geq 3$. A diagonal of a polygon is a line segment connecting any two nonadjacent vertices of the polygon. The number of diagonals D of a polygon of n sides is given by

$$D = \frac{n(n - 3)}{2}$$

How many sides does the polygon have that has 405 diagonals?

17. In exercise 15, (a) how many pens will be supplied at the equilibrium price, (b) what is the demand at that price?

18. A small manufacturer finds the total cost C for a solar energy device to be

$$C = 50x^2 - 24,000$$

and the total revenue R at a price $100 per unit to be

$$R = 100x$$

where x is the number of units manufactured and sold. What is the break-even point (that is, where total cost = total revenue) to the nearest unit?

19. As in exercise 18, a steel producer's annual cost and revenue are given by

$$C = 20 - 0.4x^2 \text{ and } R = 0.8x$$

where x is the number of units produced and sold. To the nearest unit, what is the break-even point? (See exercise 18.)

20. The profit P in dollars in the manufacture and sale of a product is given by

$$P = \frac{1}{100}n^2 - 20n$$

where n is the number of units manufactured. How many units of the product must be manufactured to make a profit of $20,000?

21. A baker makes a profit P in cents according to the equation

$$P = -n^2 + 240n$$

where n is the number of cakes baked and sold. How many cakes should be made to realize a profit of $144?

22. What is the profit if the baker makes (a) 100 cakes, (b) 120 cakes, (c) 200 cakes? Can we draw any conclusion from these results? (Use the formula given in exercise 21.)

See example 6–4 A–3.

Geometric problems

Example A rectangular plot of ground has an area of 161 square meters. If the length of the rectangle is two more than three times the width, what are the dimensions of the rectangle?

Solution Let w = the width of the rectangle then $\ell = 3w + 2$. Using $A = \ell w$,

$$161 = (3w + 2)w \qquad \text{Replace A with 161 and } \ell \text{ with } 3w + 2$$
$$161 = 3w^2 + 2w \qquad \text{Multiply in right member}$$
$$3w^2 + 2w - 161 = 0 \qquad \text{Write in standard form}$$
$$(3w + 23)(w - 7) = 0 \qquad \text{Factor left member}$$
$$3w + 23 = 0 \quad \text{or} \quad w - 7 = 0 \qquad \text{Set each factor equal to 0}$$
$$w = -\frac{23}{3} \qquad\qquad w = 7$$

$\ell = 3w + 2$

Reject $-\dfrac{23}{3}$ since the side of a rectangle cannot be negative. Then $w = 7$ and

$\ell = 3w + 2 = 3(7) + 2 = 23$. The rectangle is 7 meters wide and 23 meters long.
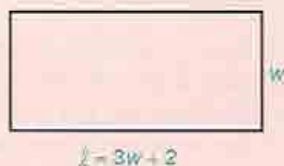
24. If a rectangular-shaped playground has a length 5 feet less than three times the width and the distance from one corner to the opposite corner is 100 feet, find the dimensions of the playground.

25. Given a rectangular plot of land whose length is 30 feet longer than its width, what are the dimensions of the plot if a diagonal path across the plot is 240 feet long? (*Hint:* The diagonal is the hypotenuse of a right triangle.)

26. The diagonal of a square piece of metal is 50 centimeters long. Find the length of each side of the piece of metal.

27. The diagonal of a square flower bed is $5\sqrt{2}$ feet. Find the length of each side.

28. The area of a rectangular floor is 24 square meters. Find the dimensions of the floor if the width is 2 meters less than the length. ($A = \ell w$.)

29. A triangular-shaped plate has an altitude that is 5 inches longer than its base. If the area of the plate is 52 square inches, what is the altitude of the triangle? $\left(A = \dfrac{1}{2}bh. \right)$

30. A rectangle that is 6 centimeters long and 3 centimeters wide has its dimensions increased by the same amount. The area of this new rectangle is three times that of the old rectangle. What are the dimensions of the new rectangle? (*Hint:* Let x be the increase in the length and width.)

31. A rectangular field is twice as long as it is wide. Find the dimensions of the field if it contains 5,000 square yards.

32. Find the length of the radius r of a circular disk whose area A is 154 square feet.

(*Note:* $A = \pi r^2$ and use $\pi = \dfrac{22}{7}$ as an approximation of π.)

33. Find the length of the diameter D of a circular gear whose area A is 12.56 square feet. (*Note:* $A = \pi r^2$ and $D = 2r$. Use $\pi = 3.14$ as an approximation of π.)

34. The base and altitude of a triangle are 3 inches and 5 inches, respectively. If the base is increased by twice as much as the altitude, the area of the new triangle is twice that of the old triangle. What are the lengths of the base and the altitude of the new triangle?

Use the Pythagorean Theorem. See example 6–4 A–2.

Right triangle problems

35. Find the length of the shortest side of a right triangle whose sides are y inches, $y + 1$ inches, and $y + 8$ inches.

36. The hypotenuse (longest side) of a right triangle is 10 millimeters long. One leg is 2 millimeters longer than the other. What are the lengths of the two legs?

37. The hypotenuse of a right triangle is 1 inch longer than the longer of the two legs. The shorter leg is 7 inches. Find the length of the hypotenuse.

38. A plot of ground has the shape of a right triangle. If the longer of the two legs is 9 dekameters longer than the shorter leg, and the hypotenuse is 8 dekameters longer than the longer leg, find the lengths of the three sides.

See example 6–4 A–4.

39. It takes Debbie 39 minutes longer to do a job than it takes Lisa to do the same job. Working together, they can do the job in 40 minutes. How long would it take each girl to do the job working alone?

40. Working together, Tom Roggenbeck and Amy Miyazaki can mow a lawn in 1 hour. Working alone, it would take Amy 90 minutes longer than it would take Tom. How long would it take Amy to mow the lawn alone?

41. A water tank has an inlet pipe (to fill the tank) and an outlet pipe (to empty the tank). The tank will fill in 8 hours when both pipes are open. If it takes 2 hours longer to empty the tank than it does to fill the tank, how long does it take the outlet pipe to empty the tank?
[*Hint:* (rate of inlet) — (rate of outlet) = (rate to fill).]

42. Tim can paint a room in 6 hours less time than Tom. If they can paint the room in 4 hours working together, how long would it take each boy to paint the room working alone?

Review exercises

1. Square $(2\sqrt{3} - 4)^2$. See section 5–6.

Find the solution set of the following equations. See sections 2–1 and 6–1.

2. $3y - 2 = 4(6y + 1)$

3. $y^2 - 5y = 24$

Simplify the following expressions. See section 5–1.

4. $(16)^{3/4}$

5. $x^{1/3} \cdot x^{1/2} \cdot x$

6. $\dfrac{x^{2/3}}{x^{1/2}}$

7. Rationalize the denominator of $\dfrac{\sqrt{2}}{\sqrt{3}}$. See section 5–4.

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6–5 ■ *Equations involving radicals*

Equations in which at least one member contains a radical expression that has a variable in the radicand are called **radical equations.** When the radicals are of the second order (involving square root), we use the methods we have learned for solving equations together with the following property to find the solution set.

— *Property of nth power* —

Given a natural number $n > 1$ and real algebraic expressions P and Q, all of the solutions of the equation

$$P = Q$$

are contained in the solution set of the equation

$$P^n = Q^n$$

Concept

If each member of an equation is raised to some natural number power greater than one, the solution set of the original equation is a subset of the solution set of the resulting equation.

From this property, we can see that the equation $P^n = Q^n$ *may* contain solutions that are not solutions of the equation $P = Q$. Such solutions are called **extraneous solutions.** To illustrate, consider the equation $x - 3 = 5$, whose solution is 8. If we square each member of the equation, we obtain

$$(x - 3)^2 = 5^2$$
$$x^2 - 6x + 9 = 25$$
$$x^2 - 6x - 16 = 0 \qquad \text{Write in standard form}$$
$$(x - 8)(x + 2) = 0$$
$$x - 8 = 0 \quad \text{or} \quad x + 2 = 0$$
$$x = 8 \quad \text{or} \qquad x = -2$$
$$S = \{8, -2\}$$

Since -2 is not a solution of the original equation, $x - 3 = 5$, we call -2 an extraneous solution.

Since the application of the *n*th power property may produce extraneous solutions, *each solution* obtained through the use of the property *must be checked* into the original equation. Now consider some radical equations where we apply this property.

■ *Example 6–5 A*

Find the solution set of each equation. Identify extraneous solutions, if they exist.

1. $\sqrt{x} - 4 = 5$
$$\sqrt{x} = 9 \qquad\qquad \text{Add 4 to each member}$$
$$(\sqrt{x})^2 = (9)^2 \qquad \text{Square each member}$$
$$x = 81$$

Check our solution in the *original* equation.

$$\sqrt{x} - 4 = 5$$
$$\sqrt{81} - 4 = 5 \qquad\qquad \text{Replace } x \text{ with 81}$$
$$9 - 4 = 5$$
$$5 = 5 \qquad\qquad \text{(True)}$$

No extraneous solutions exist and the solution set is $\{81\}$.

2. $\sqrt{2x - 5} = 3$
   $(\sqrt{2x - 5})^2 = 3^2$ Square each member
   $2x - 5 = 9$
   $2x = 14$
   $x = 7$

We must check our possible solution.

$\sqrt{2x - 5} = 3$
$\sqrt{2(7) - 5} = 3$
$\sqrt{9} = 3$
$3 = 3$ (True)

There is no extraneous solution and the solution set is $\{7\}$.

3. $\sqrt{x} = x - 12$
   $(\sqrt{x})^2 = (x - 12)^2$ Square each member
   $x = x^2 - 24x + 144$
   ⌞————————— Don't forget the middle term

Note Remember, $(x - 12)^2 = x^2 - 24x + 144$. A common error we can make is to say $(x - 12)^2 = x^2 - 12^2 = x^2 - 144$. $(x - 12)^2 \neq x^2 - 144$.

$0 = x^2 - 25x + 144$ Subtract x from each member
$0 = (x - 16)(x - 9)$
$x - 16 = 0$ or $x - 9 = 0$
$x = 16$ $x = 9$

Check:

Let $x = 16$, then   Let $x = 9$, then
$\sqrt{16} = 16 - 12$ $\sqrt{9} = 9 - 12$ Replace x with 16, 9
$4 = 16 - 12$ $3 = 9 - 12$
$4 = 4$ (True) $3 = -3$ (False)

Therefore 9 is an extraneous solution and the solution set of the equation $\sqrt{x} = x - 12$ is $\{16\}$.

4. $\sqrt{x + 4} - \sqrt{x - 3} = 1$
   When two terms contain radical expressions, we must rewrite the equation with a radical expression in each member. Add $\sqrt{x - 3}$ to each member.

$\sqrt{x + 4} = \sqrt{x - 3} + 1$
$(\sqrt{x + 4})^2 = (\sqrt{x - 3} + 1)^2$ Square each member
$x + 4 = x - 3 + 2\sqrt{x - 3} + 1$ Watch this step carefully
Then $x + 4 = x - 2 + 2\sqrt{x - 3}$

Isolate the remaining radical expression in one member by adding $-x + 2$ to both members. Then

$x + 4 - x + 2 = 2\sqrt{x - 3}$
$6 = 2\sqrt{x - 3}$ Divide each member by 2
$3 = \sqrt{x - 3}$
$(3)^2 = (\sqrt{x - 3})^2$ Square each member
$9 = x - 3$
$x = 12$

Check: Let $x = 12$, then

$$\sqrt{12 + 4} - \sqrt{12 - 3} = 1$$
$$\sqrt{16} - \sqrt{9} = 1$$
$$4 - 3 = 1$$
$$1 = 1 \quad \text{(True)}$$

There is no extraneous solution and the solution set is $\{12\}$.

5. $\sqrt[3]{2x - 3} = 2$
$(\sqrt[3]{2x - 3})^3 = 2^3$ Cube each member.
$2x - 3 = 8$
$2x = 11$
$x = \dfrac{11}{2}$

Note Extraneous roots occur only when the radical involves even roots, so there is no need to check the answer.

The solution set is $\left\{ \dfrac{11}{2} \right\}$.

6. $\sqrt{x} + 1 = 0$
$\sqrt{x} = -1$ Subtract 1 from each member.
$x = 1$ Square each member.

Check: Let $x = 1$, then

$$\sqrt{1} + 1 = 0$$
$$1 + 1 = 0$$
$$2 = 0 \quad \text{(False)}$$

Since 1 does not check, the solution set is $\emptyset$, and 1 is an extraneous solution.

▶ *Quick check* Find the solution set of $\sqrt{z} = z - 6$. Identify any extraneous solutions, if they exist.

In general, we use this procedure to solve a radical equation.

— To solve radical equations —————————————

1. Isolate one radical term alone in one member of the equation.
2. Raise each member of the equation to the power that is the same as the index of the radical.
3. Solve the resulting equation. If a radical term remains, repeat steps 1 and 2.
4. Check all possible solutions in the original equation if the equation involves an even root.

— Mastery points —————————————

Can you
 ■ Identify extraneous solutions of a radical equation?
 ■ Find the solution set of a radical equation?

Exercise 6–5

Find the solution set of each equation. Identify extraneous solutions, if they exist. See example 6–5 A.

Example $\sqrt{z} = z - 6$

Solution

$$(\sqrt{z})^2 = (z - 6)^2 \qquad \text{Square each member}$$
$$z = z^2 - 12z + 36 \qquad \text{Multiply in right member}$$
$$z^2 - 13z + 36 = 0 \qquad \text{Write equation in standard form}$$
$$(z - 9)(z - 4) = 0 \qquad \text{Factor left member}$$
$$z - 9 = 0 \quad \text{or} \quad z - 4 = 0 \qquad \text{Set each factor equal to 0}$$
$$z = 9 \qquad\qquad z = 4$$

Check:

$$z = 9 \qquad\qquad z = 4$$
$$\sqrt{9} = 9 - 6 \qquad\qquad \sqrt{4} = 4 - 6$$
$$3 = 3 \quad \text{(True)} \qquad\qquad 2 = -2 \quad \text{(False)}$$

4 is an extraneous solution and the solution set is $\{9\}$.

1. $\sqrt{x} - 6 = 3$
2. $\sqrt{x} + 9 = 13$
3. $\sqrt{x} + 5 = 2$
4. $\sqrt{x + 2} = 5$
5. $\sqrt{y - 4} = 7$
6. $\sqrt{3k - 1} - 5 = 0$
7. $\sqrt{5z + 1} - 11 = 0$
8. $\sqrt{9a + 5} = \sqrt{3a - 1}$
9. $\sqrt{2p + 5} = \sqrt{3p + 4}$
10. $\sqrt{r + 6} = \sqrt{3r + 2}$
11. $2\sqrt{3z} = \sqrt{5z + 7}$
12. $3\sqrt{x - 3} = \sqrt{2x - 5}$
13. $2\sqrt{2z - 1} = \sqrt{4z}$
14. $\sqrt{p}\,\sqrt{p - 8} = 3$
15. $\sqrt{y}\,\sqrt{y - 5} = 6$
16. $\sqrt{m}\,\sqrt{2m + 4} = 4$
17. $\sqrt{2z}\,\sqrt{z - 3} = 6$
18. $\sqrt{x^2 + 3} - 2 = 0$
19. $\sqrt{y^2 + 7} - 3 = 0$
20. $\sqrt{w^2 - 6w} = 4$
21. $\sqrt{t^2 + 8t} = 3$
22. $\sqrt{z^2 + 12} + 3 = 0$
23. $\sqrt{y^2 + 7} - 4 = 0$
24. $x\sqrt{2} = \sqrt{6 - 4x}$
25. $y\sqrt{3} = \sqrt{9y + 30}$
26. $z\sqrt{3} = \sqrt{z + 1}$
27. $t\sqrt{2} = \sqrt{5t - 2}$
28. $\sqrt{z + 12} = z$
29. $\sqrt{2x} = x - 4$
30. $p = \sqrt{6 - p}$
31. $q = \sqrt{5q - 4}$
32. $\sqrt{x - 2} = x - 2$
33. $\sqrt{u - 1} = u - 3$
34. $\sqrt{3x + 10} - 3x = 4$
35. $2t - \sqrt{5 - 2t} = 5$
36. $1 + \sqrt{5x + 9} = x$
37. $\sqrt{5x + 1} - 1 = \sqrt{3x}$
38. $\sqrt{v + 5} = 5 - \sqrt{v}$
39. $\sqrt{2n + 3} - \sqrt{n - 2} = 2$
40. $3 - \sqrt{y + 4} = \sqrt{y + 7}$
41. $\sqrt{p + 1} = \sqrt{2p + 9} - 2$
42. $(2y + 3)^{1/2} - (4y - 1)^{1/2} = 0$
   [*Hint:* $(2y + 3)^{1/2} = \sqrt{2y + 3}$.]
43. $(2x^2 + 3x - 5)^{1/2} = (2x^2 - x - 2)^{1/2}$
44. $(4x + 2)^{1/2} - (2x)^{1/2} = 0$
45. $(1 - 2y)^{1/2} + (y + 5)^{1/2} = 4$
46. $(x - 2)^{1/2} = (5x + 1)^{1/2} - 3$
47. $\sqrt[3]{x - 7} = 3$
48. $\sqrt[4]{2y - 3} = 1$
49. $\sqrt[3]{x^2 - 6x - 8} = 2$
50. $\sqrt[4]{2x^2 - 3x - 8} = -1$
51. $(4p - 3)^{1/3} = -2$
52. $(2q - 5)^{1/5} = -1$

Solve the following equations and formulas for the indicated variable. Assume all denominators are nonzero.

Example $3x\sqrt{x + y} = 2$ for y

Solution

$$(3x\sqrt{x + y})^2 = 2^2 \qquad \text{Square each member}$$
$$9x^2(x + y) = 4$$
$$9x^3 + 9x^2y = 4 \qquad \text{Perform the indicated multiplication}$$
$$9x^2y = 4 - 9x^3 \qquad \text{Subtract } 9x^3 \text{ from both members}$$
$$y = \frac{4 - 9x^3}{9x^2} \qquad \text{Divide each member by } 9x^2$$

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53. $r = \sqrt{\dfrac{A}{4\pi h}}$ for A

54. $t = \sqrt{\dfrac{2s}{g}}$ for s

55. $r = \sqrt{\dfrac{A}{\pi} - R^2}$ for A

56. $r = \sqrt{\dfrac{V}{\pi h}}$ for h

57. $D = \sqrt[3]{\dfrac{6A}{\pi}}$ for A

58. $v = \sqrt{\dfrac{2gKE}{W}}$ for W

59. At an altitude of h ft above the sea or level ground, the distance d in miles that a person can see an object is given by

$$d = \sqrt{\dfrac{3h}{2}}$$

How tall must a person be to see an object 3 miles away?

60. The formula for approximating the velocity V in miles per hour of a car based on the length of its skid marks S (in feet) on dry pavement is given by

$$V = 2\sqrt{6S}$$

If the velocity is 48 mph, how long will the skid marks be?

61. On wet pavement, the formula in exercise 60 is given by

$$V = 2\sqrt{3S}$$

How long will the skid marks be if the car is traveling at 30 mph on wet pavement?

62. Find the number whose principal square root is $3i$. (*Hint:* Let $\sqrt{x} = 3i$.)

63. Find the number whose principal third root is -3.

64. Find the number whose principal fourth root is 4.

Review exercises

Find the solution set of the following equations. See sections 6–1 and 6–3.

1. $y^2 + 6y = 16$

2. $3x^2 + 2x - 2 = 0$

Find the solution set of the following inequalities. See section 2–5.

3. $4y - 1 \geq 2y + 7$

4. $-4 < 3x + 2 \leq 5$

5. A number plus one-half the number plus one-third the number is 33. Find the number. See section 2–3.

6. Write an inequality to state that the temperature on a given day had a low of 21° and a high of 62°. See section 2–5.

6–6 ■ Equations that are quadratic in form

There are a number of equations that are not quadratic equations but they can, nevertheless, be written in **quadratic form**

$$au^2 + bu + c = 0, \qquad a > 0$$

and solved as we solved quadratic equations. The variable u in the equation represents some expression in another variable. Such equations are said to be reducible to quadratic equations by making a substitution. We then solve the resulting equation by the methods of this chapter.

■ *Example 6–6 A*

Find the solution set of each equation.

1. $x^4 + 4x^2 - 12 = 0$

$$x^4 \quad + 4x^2 \quad - 12 = 0$$
$$\downarrow \qquad \downarrow \qquad \downarrow \qquad \downarrow$$
$$(x^2)^2 + 4(x^2) - 12 = 0 \qquad \text{Replace } x^4 \text{ with } (x^2)^2$$
$$\downarrow \qquad \downarrow \qquad \downarrow \qquad \downarrow$$
$$u^2 \quad + 4u \quad - 12 = 0 \qquad \text{Replace } x^2 \text{ with } u \text{ to make factoring easier}$$

Solve the equation $u^2 + 4u - 12 = 0$.

$$(u + 6)(u - 2) = 0 \qquad\qquad \text{Factor the left member}$$
$$u + 6 = 0 \quad \text{or} \quad u - 2 = 0 \qquad \text{Set each factor equal to 0}$$
$$u = -6 \qquad\qquad u = 2$$

THESE ARE *NOT* THE SOLUTIONS FOR x. We *must* now replace u with x^2 to get back to the original equation.

$$x^2 = -6 \qquad \text{or} \quad x^2 = 2 \qquad \text{Replace } u \text{ with } x^2$$
$$x = \pm\sqrt{-6} \qquad\qquad x = \pm\sqrt{2} \qquad \text{Extract the roots}$$
$$x = \pm i\sqrt{6} \qquad\qquad\qquad\qquad \sqrt{-6} = i\sqrt{6}$$

The solution set is $\{i\sqrt{6}, -i\sqrt{6}, \sqrt{2}, -\sqrt{2}\}$.

Note The equation is of fourth degree, and we obtained four solutions. The degree of an equation indicates the *maximum* number of solutions we can expect to find.

2. $y + 3\sqrt{y} - 4 = 0$

$$y \qquad + 3\sqrt{y} - 4 = 0$$
$$\downarrow \qquad\quad \downarrow \qquad \downarrow$$
$$(\sqrt{y})^2 + 3\sqrt{y} - 4 = 0 \qquad \text{Replace } y \text{ with } (\sqrt{y})^2$$
$$\downarrow \qquad\quad \downarrow \qquad \downarrow$$
$$u^2 \quad + 3u \quad - 4 = 0 \qquad \text{Replace } \sqrt{y} \text{ with } u$$

Note We now have a quadratic equation and will have, at most, two solutions.

$$(u + 4)(u - 1) = 0 \qquad\qquad \text{Factor the left member}$$
$$u = -4 \text{ or } u = 1$$

THESE ARE *NOT* THE SOLUTIONS FOR y. We *must* now replace u with $\sqrt{y}$.

$$\sqrt{y} = -4 \qquad \text{or} \qquad \sqrt{y} = 1$$
$$(\sqrt{y})^2 = (-4)^2 \qquad\qquad (\sqrt{y})^2 = (1)^2 \qquad \text{Square each member}$$
$$y = 16 \qquad\qquad\qquad y = 1$$

Since we squared each member of each equation to get these *possible* solutions, we *must* check the original equation $y + 3\sqrt{y} - 4 = 0$ for extraneous solutions.

Check:

Let $y = 16$, then

$$16 + 3\sqrt{16} - 4 = 0$$
$$16 + 3(4) - 4 = 0$$
$$16 + 12 - 4 = 0$$
$$28 - 4 = 0$$
$$24 = 0 \quad \text{(False)}$$

Let $y = 1$, then

$$1 + 3\sqrt{1} - 4 = 0$$
$$1 + 3 - 4 = 0$$
$$4 - 4 = 0$$
$$0 = 0 \quad \text{(True)}$$

Thus 16 is an extraneous solution and the solution set is $\{1\}$.

Note We could have predicted that 16 would be extraneous since it came from the equation $\sqrt{y} = -4$ and the principal square root of a number is *never* negative.

3. $3x^{-2} + 7x^{-1} - 6 = 0$

$$3x^{-2} \quad + 7x^{-1} - 6 = 0$$
$$\downarrow \qquad\quad \downarrow \qquad \downarrow$$
$$3(x^{-1})^2 + 7x^{-1} - 6 = 0 \qquad\qquad \text{Replace } x^{-2} \text{ with } (x^{-1})^2$$
$$\downarrow \qquad\qquad \downarrow \qquad \downarrow$$
$$3u^2 \quad\quad + 7u \quad - 6 = 0 \qquad\qquad \text{Replace } x^{-1} \text{ with } u$$

$$(3u - 2)(u + 3) = 0$$
$$3u - 2 = 0 \quad \text{or} \quad u + 3 = 0$$
$$u = \frac{2}{3} \qquad\qquad u = -3$$

THESE ARE *NOT* THE SOLUTIONS FOR x. We *must* replace u with x^{-1} or $\dfrac{1}{x}$.

$$\frac{1}{x} = \frac{2}{3} \quad \text{or} \qquad \frac{1}{x} = -3 \qquad\qquad \text{Replace } u \text{ with } \frac{1}{x}$$
$$2x = 3 \qquad\qquad -3x = 1 \qquad\qquad \text{Multiply by the LCD}$$
$$x = \frac{3}{2} \qquad\qquad x = -\frac{1}{3}$$

The solution set is $\left\{ \dfrac{3}{2}, -\dfrac{1}{3} \right\}$.

Note This equation could be written $\dfrac{3}{x^2} + \dfrac{7}{x} - 6 = 0$ and solved as a rational equation.

Note To recognize each of these as being the quadratic type, you must observe the characteristic that each of these equations has in common. Notice that the square of the variable factor of the middle term yields the variable factor of the first term. That is,

1. $(x^2)^2 = x^4$
2. $(\sqrt{y})^2 = y$
3. $(x^{-1})^2 = x^{-2}$

▶ *Quick check* Find the solution set of $5z^{-2} + 6z^{-1} - 8 = 0$. ■

Mastery points

Can you
- Identify a quadratic-type equation?
- Solve any quadratic-type equation?

Exercise 6–6

Find the solution set of each equation. Identify extraneous solutions, if they exist. See example 6–6 A.

Example $5z^{-2} + 6z^{-1} - 8 = 0$

Solution $5z^{-2} \quad + 6z^{-1} - 8 = 0$

$5(z^{-1})^2 + 6z^{-1} - 8 = 0$ Replace z^{-2} with $(z^{-1})^2$

$5u^2 \quad + 6u \quad - 8 = 0$ Replace z^{-1} with u

We now solve the equation $5u^2 + 6u - 8 = 0$.

$(5u - 4)(u + 2) = 0$ Factor the left member

$5u - 4 = 0 \quad$ or $\quad u + 2 = 0$ Set each factor equal to 0

$u = \dfrac{4}{5} \qquad\qquad u = -2$

Replace u with z^{-1} or $\dfrac{1}{z}$.

$\dfrac{1}{z} = \dfrac{4}{5} \quad$ or $\quad \dfrac{1}{z} = -2$ Replace u with $\dfrac{1}{z}$

$4z = 5 \qquad\qquad -2z = 1$ Multiply by the LCD

$z = \dfrac{5}{4} \qquad\qquad z = -\dfrac{1}{2}$

The solution set is $\left\{ -\dfrac{1}{2}, \dfrac{5}{4} \right\}$.

---

1. $x^4 - 6x^2 + 5 = 0$
2. $y^4 + 3y^2 - 28 = 0$
3. $3z^4 + z^2 = 2$
4. $4p^4 = 25p^2 - 6$
5. $(x - 2)^4 + 9(x - 2)^2 + 8 = 0$
6. $(m + 5)^4 - 4(m + 5)^2 + 4 = 0$
7. $(p^2 - 2p)^2 - 7(p^2 - 2p) - 8 = 0$
8. $(x + 3)^2 - 6(x + 3) + 8 = 0$
9. $q - 6\sqrt{q} - 27 = 0$
10. $x + \sqrt{x} = 12$
11. $2y + 3\sqrt{y} + 1 = 0$
12. $3t - 4\sqrt{t} = 4$
13. $p - 5p^{1/2} = -4$
14. $2x = 9 - 3x^{1/2}$
15. $(x^2 - 3) - 3\sqrt{x^2 - 3} - 28 = 0$
16. $(m^2 + 1) + \sqrt{m^2 + 1} = 20$
17. $(y - 6) - \sqrt{y - 6} - 2 = 0$
18. $(z + 1) - 8\sqrt{z + 1} + 7 = 0$
19. $y^{2/3} + 3y^{1/3} - 10 = 0$
20. $2y^{2/3} - 3y^{1/3} = 2$
21. $5p^{2/3} + 11p^{1/3} + 2 = 0$
22. $u^{3/2} - 7u^{3/4} - 8 = 0$
23. $x^{3/2} - 2x^{3/4} + 1 = 0$
24. $p^{3/4} - 3p^{3/8} + 2 = 0$
25. $z^{-2} - z^{-1} - 12 = 0$
26. $r^{-2} - 7r^{-1} + 6 = 0$
27. $2y^{-2} = 9y^{-1} - 4$
28. $x^{-4} = 5x^{-2} - 4$
29. $4y^{-4} + 4 = 17y^{-2}$
30. $(t^2 - t)^2 - 4(t^2 - t) - 12 = 0$
31. $(x^2 + 3x)^2 - 8(x^2 + 3x) = 20$

32. Given the equation $x - 2\sqrt{x} - 15 = 0$, find the solution set by the method in (a) section 6–5 and (b) section 6–6.

33. Find the solution set of the equation $y + 4\sqrt{y} - 12 = 0$ using the method of (a) section 6–5 and (b) section 6–6.

Review exercises

Perform the indicated operations. See sections 4–2 and 4–3.

1. $\dfrac{3x}{x^2 - 4} - \dfrac{4x}{x - 2}$

2. $\dfrac{3x + 1}{x - 4} \div \dfrac{3x^2 - 2x - 1}{x^2 - 16}$

3. Given $y = 5x - 2$, find y when (a) $x = 2$, (b) $x = -5$, (c) $x = 0$. See section 1–5.

4. Find the solution set of $4x - 3 \geq 2(x + 1)$. See section 2–5.

Find the solution set of the following. See sections 2–4 and 2–6.

5. $|2x - 3| = 5$

6. $|x - 3| \leq 4$

7. $|3x + 1| > 2$

6-7 ■ Quadratic and rational inequalities

We have solved linear inequalities in chapter 2 and quadratic equations in this chapter. The methods learned there are now used to solve **quadratic inequalities.**

A quadratic inequality is any inequality that can be written in the form

$$ax^2 + bx + c < 0, \qquad ax^2 + bx + c > 0,$$
$$ax^2 + bx + c \leq 0, \quad \text{or} \quad ax^2 + bx + c \geq 0,$$

where a, b, and c are real numbers, $a > 0$.

The method used to solve quadratic inequalities is shown in the following example. Given

$$x^2 + 2x - 3 > 0$$

factor the left member to get

$$(x + 3)(x - 1) > 0$$

We now set each factor in the left member equal to 0 and solve each equation.

$$x + 3 = 0 \quad \text{or} \quad x - 1 = 0$$
$$x = -3 \qquad\qquad x = 1$$

The roots -3 and 1, called *critical numbers*, divide the real number line into three regions as shown in figure 6–1.
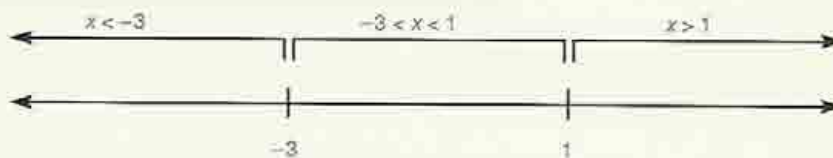
Figure 6–1

It can be shown that if one number in a given region makes the product $(x + 3)(x - 1)$ positive (as we want it to be), then all numbers in that region will make the product positive.

We now choose a number within each region as a *test number* to see if it satisfies the inequality. Suppose we choose the numbers

$$-4 \text{ in } x < -3; \quad 0 \text{ in } -3 < x < 1; \quad 2 \text{ in } x > 1$$

Note *Never* test a region with the test number as a critical number.

Substitute these numbers into the inequality.

$$\text{When } x = -4, \text{ then } (x + 3)(x - 1) = (-4 + 3)(-4 - 1)$$
$$= (-1)(-5) = 5 \quad \text{Positive}$$
$$\text{When } x = 0, \text{ then } (x + 3)(x - 1) = (0 + 3)(0 - 1)$$
$$= (3)(-1) = -3 \quad \text{Negative}$$
$$\text{When } x = 2, \text{ then } (x + 3)(x - 1) = (2 + 3)(2 - 1)$$
$$= (5)(1) = 5 \quad \text{Positive}$$

Thus the solution set includes all points in the regions $x < -3$ or $x > 1$. The solution set of the inequality $x^2 + 2x - 3 > 0$ is

$$\{x \mid x < -3 \text{ or } x > 1\}$$

The solution set is graphed on the number line in figure 6–2.
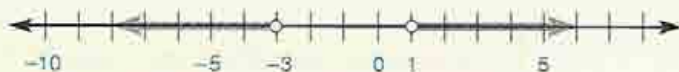
Figure 6–2

To solve a quadratic inequality

1. Write the inequality in the form

$$ax^2 + bx + c < 0, \qquad ax^2 + bx + c > 0,$$
$$ax^2 + bx + c \leq 0, \quad \text{or} \quad ax^2 + bx + c \geq 0$$

2. Factor the quadratic trinomial $ax^2 + bx + c$. If not factorable, then set $ax^2 + bx + c = 0$ and solve for x.

3. Find the *critical numbers* by setting each factor equal to 0 and solving for x.

4. Divide the number line into regions using the critical numbers. Write the regions using the same inequality symbol as the original problem.

5. Choose a *test number* (other than a critical number) within each region obtained and check the *sign* of the product at the test number.

6. Choose the region(s) that satisfy the conditions of the original inequality.

■ *Example 6–7 A*

Find the solution set of the following quadratic inequalities. Graph the solution set on the number line.

1. $x^2 - x - 6 \leq 0$

$(x - 3)(x + 2) \leq 0$ Factor the left member

$x - 3 = 0$ $x + 2 = 0$ Set each factor equal to 0

$x = 3$ $x = -2$ Solve each equation

The critical numbers are -2 and 3. We write the regions shown here using the same inequality symbols.
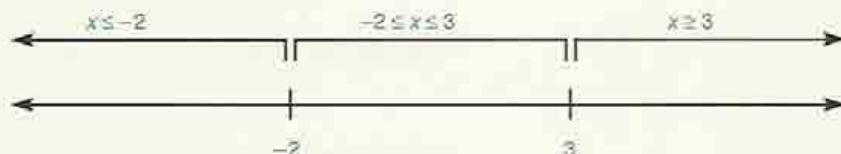
Choose the test numbers -3 in $x < -2$; 0 in $-2 < x < 3$; 4 in $x > 3$. Substitute the test numbers into the factors of the inequality $(x - 3)(x + 2) \leq 0$.

If $x = -3$, then $(x - 3)(x + 2) = (-3 - 3)(-3 + 2)$
$= (-6)(-1) = 6$ Positive

If $x = 0$, then $(x - 3)(x + 2) = (0 - 3)(0 + 2)$
$= (-3)(2) = -6$ Negative

If $x = 4$, then $(x - 3)(x + 2) = (4 - 3)(4 + 2)$
$= (1)(6) = 6$ Positive

Since we want those values of x that make the quadratic less than (negative) or equal to zero, the numbers in the region $-2 \leq x \leq 3$ satisfy the inequality. The solution set is $\{x | -2 \leq x \leq 3\}$.
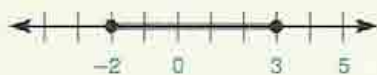
2. $3x^2 - 4x - 4 > 0$

$(3x + 2)(x - 2) > 0$ Factor the left member

$3x + 2 = 0$ $x - 2 = 0$ Set each factor equal to 0

$x = -\dfrac{2}{3}$ $x = 2$ Solve each equation

The critical numbers are $-\dfrac{2}{3}$ and 2. We write the regions as shown here.
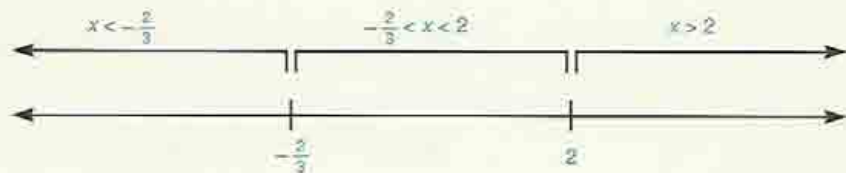
Choose the test numbers -1 in $x < -\dfrac{2}{3}$; 0 in $-\dfrac{2}{3} < x < 2$; 3 in $x > 2$.

Substitute these test numbers in the inequality $(3x + 2)(x - 2) > 0$.

If $x = -1$, then $(3x + 2)(x - 2) = [3(-1) + 2](-1 - 2)$
$$= (-1)(-3) = 3 \quad \text{Positive}$$

If $x = 0$, then $(3x + 2)(x - 2) = [3(0) + 2](0 - 2)$
$$= (2)(-2) = -4 \quad \text{Negative}$$

If $x = 3$, then $(3x + 2)(x - 2) = [3(3) + 2](3 - 2)$
$$= (11)(1) = 11 \quad \text{Positive}$$

The solution set is $\left\{ x \,\middle|\, x < -\dfrac{2}{3} \text{ or } x > 2 \right\}$.

▶ *Quick check*   Find and graph the solution set of $y^2 + 2y - 8 \le 0$.

We can apply this same method in solving a *rational* inequality such as

$$\frac{z}{z - 5} \ge 3$$

Our first inclination is to multiply each member by the denominator to clear the denominator. However we do not know whether the denominator represents a *positive* or a *negative* number. Recall that this affects the order symbol involved. An easier approach is the method used in the following example.

3. $\dfrac{z}{z - 5} \ge 3$

Write the corresponding equation and solve it.

$$\frac{z}{z - 5} = 3 \qquad \text{Change } \ge \text{ to } =$$
$$z = 3(z - 5) \qquad \text{Multiply each member by } z - 5$$
$$z = 3z - 15 \qquad \text{Solve the equation}$$
$$-2z = -15$$
$$z = \frac{15}{2}$$

Set the denominator equal to zero and solve the equation.

$$z - 5 = 0$$
$$z = 5$$

The critical numbers are 5 and $\dfrac{15}{2}$. They define these regions.
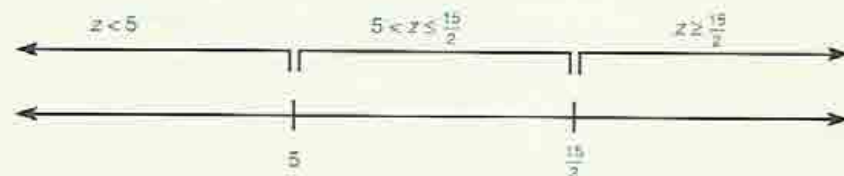
Choose the test numbers 0 in $z < 5$; 6 in $5 < z < \dfrac{15}{2}$; 8 in $z > \dfrac{15}{2}$.

Note   Since $z - 5$ is in the denominator, then $z \neq 5$ and we use $z < 5$ rather than $z \leq 5$.

Test the numbers in the given inequality.

$$\text{If } z = 0, \text{ then } \frac{(0)}{(0) - 5} = 0 \geq 3 \text{ is false.}$$

$$\text{If } z = 6, \text{ then } \frac{(6)}{(6) - 5} = 6 \geq 3 \text{ is true.}$$

$$\text{If } z = 8, \text{ then } \frac{(8)}{(8) - 5} = \frac{8}{3} \geq 3 \text{ is false.}$$

The solution set is $\left\{ z \mid 5 < z \leq \dfrac{15}{2} \right\}$.

To solve a rational inequality

1. Write the inequality as an equation and solve it.
2. Set the denominator equal to zero and solve it.
3. The solutions from steps 1 and 2 are the critical numbers. Use these numbers to divide the number line into regions.
4. Test a number in each region by substituting it into the original inequality to determine the region(s) that satisfy it.
5. Exclude any numbers that make the denominator equal to zero.

▶ *Quick check*   Find and graph the solution set. $\dfrac{x}{x + 3} \leq 2$

Mastery points

Can you
■ Find the solution set of a quadratic inequality?
■ Graph the solution set of a quadratic inequality?
■ Find and graph the solution set of a rational inequality?

Exercise 6–7

Find the solution set and graph the solution set for each of the following inequalities. Write the answer in set-builder notation. See example 6–7 A–1 and 2.

Example $y^2 + 2y - 8 \leq 0$

Solution $(y + 4)(y - 2) \leq 0$ Factor the left member

$y + 4 = 0$ $y - 2 = 0$ Set each factor equal to 0

$y = -4$ $y = 2$ Solve each equation

The critical numbers are -4 and 2. They define the regions shown here.
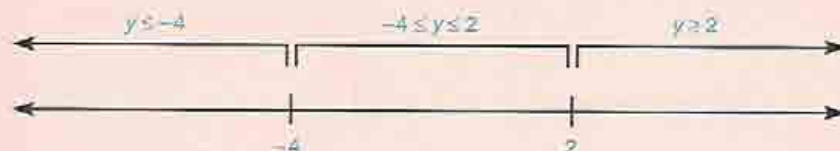
Choose the test numbers -5 in $y < -4$; 0 in $-4 < y < 2$; 3 in $y > 2$.

If $y = -5$, then $(y + 4)(y - 2) = (-5 + 4)(-5 - 2)$

$= (-1)(-7) = 7$ Positive

If $y = 0$, then $(y + 4)(y - 2)$ $= (0 + 4)(0 - 2)$

$= (4)(-2) = -8$ Negative

If $y = 3$, then $(y + 4)(y - 2)$ $= (3 + 4)(3 - 2)$

$= (7)(1) = 7$ Positive

Since we want ≤ 0 (negative or zero), the solution set is $\{y \mid -4 \leq y \leq 2\}$.

1. $(x + 3)(x - 1) > 0$ 2. $(y - 4)(y - 5) \geq 0$ 3. $(p + 7)(p + 2) \leq 0$ 4. $(z - 6)(z + 8) < 0$

5. $r(r - 1) \geq 0$ 6. $q(3q + 4) < 0$ 7. $x^2 - 5x + 4 < 0$ 8. $m^2 + 6m + 5 \geq 0$

9. $q^2 - 3q \leq 18$ 10. $t^2 + t > 30$ 11. $x^2 + 2 < 3x$ 12. $w^2 - 8 \leq 2w$

13. $u^2 \geq 12 - 4u$ 14. $x^2 - 1 < 0$ 15. $y^2 - 4 > 0$ 16. $p^2 \geq 5$

17. $5x^2 - 15 < 0$ 18. $6y^2 \geq 42$ 19. $x^2 - 5x < 0$ 20. $2m^2 - 3m < 0$

21. $2w^2 \leq -6w$ 22. $2x^2 - 7x - 4 > 0$ 23. $2y^2 + y - 6 < 0$ 24. $4z^2 + 7z + 3 \leq 0$

25. $3w^2 + 16w + 5 \geq 0$ 26. $2p^2 - 3p < 9$ 27. $9v^2 - 8 < 6v$ 28. $6w^2 - 7 \leq -11w$

29. $5y^2 - y \geq 4$ 30. $(x - 3)(x + 1)(x - 2) > 0$ 31. $(y + 4)(y - 5)(y + 6) < 0$

32. $r(3r + 5)(r - 6) \leq 0$ 33. $t(t + 5)(3t - 8) \geq 0$

Find the solution set of the following rational inequalities. Graph the solution set. Write the answer in set-builder notation. See example 6–7 A–3.

Example $\dfrac{x}{x+3} \le 2$

Solution Write the inequality as an equation and solve the equation.

$$\dfrac{x}{x+3} = 2 \qquad \text{Replace} \le \text{with} =$$
$$x = 2(x+3) \qquad \text{Multiply each member by } x+3$$
$$x = 2x + 6$$
$$x = -6$$

Set the denominator equal to zero and solve the equation.

$$x + 3 = 0$$
$$x = -3$$

The critical numbers are -6 and -3. They define the following regions.

$$x \le -6 \qquad\qquad -6 \le x < -3 \qquad\qquad x > -3$$
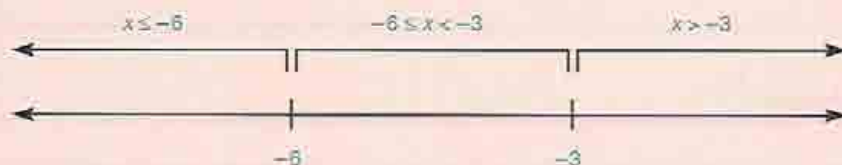
Choose the test numbers -7 in $x < -6$; -5 in $-6 < x < -3$; and 0 in $x > -3$.

If $x = -7$, then $\dfrac{(-7)}{(-7)+3} = \dfrac{-7}{-4} = \dfrac{7}{4} \le 2$ (True)

If $x = -5$, then $\dfrac{(-5)}{(-5)+3} = \dfrac{-5}{-2} = \dfrac{5}{2} \le 2$ (False)

If $x = 0$, then $\dfrac{(0)}{(0)+3} = 0 \le 2$ (True)

The solution set is $\{x \mid x \le -6 \text{ or } x > -3\}$.

Note Since $x \ne -3$, we must use $x > -3$ instead of $x \ge -3$.

34. $\dfrac{x-4}{x-2} \le 0$

35. $\dfrac{p+3}{p+1} \ge 0$

36. $\dfrac{3q-2}{2q+5} > 0$

37. $\dfrac{5t+4}{7t-3} < 0$

38. $\dfrac{1}{x} > 2$

39. $\dfrac{5}{y} < 3$

40. $\dfrac{3}{u+4} \ge 2$

41. $\dfrac{-5}{v-6} \le 1$

42. $\dfrac{1}{2r-3} < -3$

43. $\dfrac{-4}{3t+8} > 1$

44. $\dfrac{-1}{2x-7} < -2$

45. $\dfrac{x}{x+5} > 3$

46. $\dfrac{y}{y-3} > 4$

47. $\dfrac{x+6}{x-7} \le 4$

48. $\dfrac{y-4}{2y+5} < -1$

49. $\dfrac{2z-5}{z+2} > 1$

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50. $\dfrac{2p}{p-2} \le p$ **51.** $\dfrac{q}{q+4} \ge 2q$ **52.** $\dfrac{3r}{2r+1} > -2r$ **53.** $\dfrac{5t}{3t-2} < -3t$

Review exercises

1. Solve the formula $A = 2\pi r^2$ for $r > 0$.
See section 6–1.

2. Simplify the complex rational expression $\dfrac{x - \dfrac{4}{x}}{1 + \dfrac{2}{x}}$.

See section 4–4.

3. Find the solution set of the equation $x + \dfrac{2}{x} = 3$.
See section 4–6.

4. Given $y = 4x - 3$, find y when (a) $x = 5$,
(b) $x = -2$, (c) $x = 0$. See section 1–5.

5. Evaluate the rational expression $\dfrac{a-b}{c-d}$ when $a = 2$,
$b = -3$, $c = -1$, and $d = 2$. See section 1–5.

6. The width of a rectangle is 3 meters less than the length. If the rectangle has perimeter 110 meters, find the dimensions of the rectangle. See section 2–3.

Chapter 6 lead-in problem

The University of Minnesota wishes to set up a rectangular botanical garden. They have 300 meters of fence to enclose 5,000 square meters for the garden. What are the dimensions of the garden?

Solution

Since the perimeter of a rectangle is given by

$$\text{perimeter} = 2(\text{length}) + 2(\text{width})$$

and the perimeter in this case is 300 meters, then

$$300 = 2(\text{length}) + 2(\text{width})$$
$$150 = \text{length} + \text{width} \quad \text{Divide each term by 2}$$

Let x represent the length of the rectangle, then $150 - x$ represents the width of the rectangle. Using Area of a rectangle = length · width, where the area is 5,000 square meters, we substitute to obtain the equation

$$5,000 = x(150 - x)$$
$$5,000 = 150x - x^2 \quad \text{Distribute in right member}$$

$$x^2 - 150x + 5,000 = 0 \quad \text{Write equation in standard form}$$

$$(x - 100)(x - 50) = 0 \quad \text{Factor the left member}$$

$$x - 100 = 0 \quad \text{or} \quad x - 50 = 0$$
$$x = 100 \qquad\qquad x = 50$$

When the length is 100, the width is $150 - x = 150 - 100 = 50$. When the length is 50, the width is $150 - x = 150 - 50 = 100$. The dimensions of the rectangular garden are 50 meters by 100 meters.

Chapter 6 summary

1. The **standard form** of a **quadratic equation** in one variable is given by
$$ax^2 + bx + c = 0, a > 0$$

2. To solve a quadratic equation by factoring, we use the property $P \cdot Q = 0$, if and only if $P = 0$ or $Q = 0$, where P and Q are polynomials.

3. If $x^2 = p$, then $x = \pm\sqrt{p}$.

4. The quadratic equation $ax^2 + bx + c = 0$ can be solved by **completing the square** by writing the equation in the form $(x + k)^2 = d$.

5. The quadratic equation $ax^2 + bx + c = 0$ can be solved using the **quadratic formula**
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

6. Given the equation $P = Q$, where P and Q are polynomials, the solution set of the equation $P = Q$ is a subset of the solution set of the equation $P^n = Q^n$, where n is a positive integer.

7. Solutions of the equation $P^n = Q^n$, where n is a positive integer, that *are not* solutions of the equation $P = Q$ are called **extraneous solutions.**

8. A **quadratic inequality** is any inequality written in the form
$$ax^2 + bx + c < 0, ax^2 + bx + c > 0,$$
$$ax^2 + bx + c \le 0, \text{ or } ax^2 + bx + c \ge 0, \text{ where}$$
$a \ne 0$.

9. A **rational inequality** is an inequality containing at least one term that is a rational expression.

Chapter 6 error analysis

1. Solving quadratic equations
Example: $5x^2 + 25x = 0$
$$5x(x + 5) = 0$$
The solution set is $\{-5, 0, 5\}$.
Correct answer: The solution set is $\{-5, 0\}$.
What error was made? (*see page 260*)

2. Solving a quadratic equation by extracting the roots
Example: $(2x - 1)^2 = 7$
$$2x - 1 = \sqrt{7}$$
$$2x = 1 + \sqrt{7}$$
$$x = \frac{1 + \sqrt{7}}{2} \qquad \left[\frac{1 + \sqrt{7}}{2}\right]$$

Correct answer: The solution set is $\left\{\dfrac{1 - \sqrt{7}}{2}, \dfrac{1 + \sqrt{7}}{2}\right\}$.
What error was made? (*see page 262*)

3. Solve a quadratic equation by completing the square
Example:
$$x^2 + 2x - 5 = 0$$
$$x^2 + 2x = 5$$
$$x^2 + 2x + 1 = 5$$
$$(x + 1)^2 = 5$$
$$x + 1 = \pm\sqrt{5}$$
$$x = -1 \pm \sqrt{5}$$
$$\{-1 - \sqrt{5}, -1 + \sqrt{5}\}$$
Correct answer: $\{-1 - \sqrt{6}, -1 + \sqrt{6}\}$
What error was made? (*see page 268*)

4. Identifying a, b, and c of a quadratic equation
Example: $3x^2 - 4 = 2x; a = 3, b = 2, c = -4$
Correct answer: $a = 3, b = -2, c = -4$
What error was made? (*see page 272*)

5. Solving radical equations
Example: $\sqrt{x + 1} = 5 - \sqrt{x - 4}$
$$(\sqrt{x + 1})^2 = (5 - \sqrt{x - 4})^2$$
$$x + 1 = 25 - x + 4$$
$$2x = 28$$
$$x = 14 \qquad \{14\}$$
Correct answer: $\{8\}$
What error was made? (*see page 286*)

6. Solving quadratic-type equations
Example: $x^4 - 3x^2 + 2 = 0$, let $u = x^2$
$$u^2 - 3u + 2 = 0$$
$$(u - 1)(u - 2) = 0$$
$$u - 1 = 0 \quad \text{or} \quad u - 2 = 0$$
$$u = 1 \qquad\qquad u = 2 \quad \{1, 2\}$$
Correct answer: $\{-1, 1, -\sqrt{2}, \sqrt{2}\}$
What error was made? (*see page 290*)

7. Solving rational inequalities
Example: $\dfrac{x - 2}{x + 3} \le 2$

a. $\dfrac{x - 2}{x + 3} = 2$ b. $x + 3 = 0$
$$x - 2 = 2(x + 3) \qquad\qquad x = -3$$
$$x - 2 = 2x + 6$$
$$-8 = x$$
Critical numbers are -8 and -3. Test numbers are -9 in $x \le -8$, -7 in $-8 \le x \le -3$, and 0 in $x \ge -3$.
If $x = -9$, $\dfrac{-9 - 2}{-9 + 3} = \dfrac{11}{6} \le 2$ (true)

If $x = -7$, $\dfrac{-7 - 2}{-7 + 3} = \dfrac{9}{4} \le 2$ (false)

If $x = 0$, $\dfrac{0 - 2}{0 + 3} = -\dfrac{2}{3} \le 2$ (true)

The solution set is $\{x | x \le -8 \text{ or } x \ge -3\}$.
Correct answer: $\{x | x \le -8 \text{ or } x > -3\}$
What error was made? (*see page 296*)

8. Factoring the sum of two squares
   Example: $x^2 + 49 = (x)^2 + (7)^2 = (x + 7)^2$
   Correct answer: $x^2 + 49$ is not factorable in the set of real numbers.
   What error was made? (see page 142)

9. Product of two complex numbers
   Example: $\sqrt{-2} \cdot \sqrt{-8} = \sqrt{-2 \cdot -8} = \sqrt{16} = 4$
   Correct answer: -4
   What error was made? (see page 250)

10. Solving quadratic equations using the quadratic formula
    Example: Find the solution set of $x^2 - 2x - 7 = 0$ using $a = 1, b = -2, c = -7$.
    $$x = -(-2) + \frac{\sqrt{(-2)^2 - 4(1)(-7)}}{2(1)}$$
    $$= 2 + \frac{\sqrt{30}}{2}$$
    The solution set is $\left\{ \frac{4 - \sqrt{30}}{2}, \frac{4 + \sqrt{30}}{2} \right\}$
    Correct answer: $\left\{ \frac{2 - \sqrt{30}}{2}, \frac{2 + \sqrt{30}}{2} \right\}$
    What error was made? (see page 272)

Chapter 6 critical thinking

Which integers can be written as the sum of 4 consecutive integers?

Example $18 = 3 + 4 + 5 + 6$

Chapter 6 review

[6-1]

Find the solution set of the following quadratic equations by factoring or extracting the roots.

1. $x^2 - 9x - 10 = 0$
2. $y^2 - 4y = 32$
3. $4p^2 = 7p$
4. $9x^2 - 36 = 0$
5. $4z^2 - 12z + 5 = 0$
6. $2n^2 = 3n + 5$
7. $\frac{y}{2} + \frac{1}{3y} = \frac{7}{6}$
8. $(5x - 1)(x + 3) - 2x(2x + 6) = 0$
9. $(m + 7)^2 = 64$
10. $(3z - 4)^2 = 16$

11. A projectile is fired vertically upward with an initial velocity of 640 ft/sec. The distance S (in feet) above the ground after t seconds is given by
$$S = -16t^2 + 640t$$
When will the projectile reach a height of 1,600 feet? When will the projectile hit the ground?

[6-2]

Find the solution set of the given quadratic equations by completing the square.

12. $y^2 + 3y - 8 = 0$
13. $2p^2 - 3p + 1 = 0$
14. $5p^2 + 3p = -1$
15. $(4p + 3)(2p - 1) = p(p + 3)$
16. $\frac{2}{3}x^2 - \frac{1}{4}x + 1 = 0$

[6-3]

Find the solution set of the given quadratic equation by using the quadratic formula.

17. $x^2 - 11x + 10 = 0$
18. $3y^2 + 3y - 2 = 0$
19. $2z^2 - 3 = 1$
20. $2z^2 = -z - 4$
21. $3x - 2 = \frac{5}{x}$
22. $(2y - 3)^2 = 3y - 2$

Solve the following word problems.

23. Solve the equation $4x^2 + xy - 2y^2 = 0$ for x in terms of y using the quadratic formula, where y is positive.

24. A beam of length L has a maximum displacement at a distance x from the end. This is expressed by the equation

$$2x^2 - 3xL + L^2 = 0$$

Solve the equation for x by using the quadratic formula.

[6–4]

Solve the following by using quadratic equations.

25. An object fired vertically into the air with an initial velocity v_0 feet per second will be at a distance h feet in the air at t seconds after launching, according to the equation

$$h = v_0 t - 16t^2$$

How long will it take the object to reach a height of 75 feet if the initial velocity is 120 feet per second (to the nearest tenth)?

26. A rectangular solar panel has an area of 1.0 square meter. If the length is 150 centimeters more than the width, find the dimensions of the panel. (*Hint:* Use $A = \ell w$ and 1 meter = 100 centimeters.)

27. The shortest leg of a right triangular brace is 9.0 feet shorter than the hypotenuse and 7.0 feet shorter than the other leg. What are the lengths of the sides of the brace?

28. Mary can paint the exterior of a house in 2 hours less time than Dick can. Working together, they can paint the house in 12 hours. How long would it take each person to paint the house working alone?

[6–5]

Find the solution set of each radical equation. Indicate any extraneous solutions.

29. $y = \sqrt{y + 20}$

30. $\sqrt{3x + 5} + 1 = 3x$

31. $\sqrt{y + 2} + 1 = \sqrt{y + 6}$

32. $\sqrt{x^2 - x - 5} = 1$

33. $\sqrt{5x} + 2x = \sqrt{20x + 5}$

34. The radius r of a sphere is determined by the equation

$$r = \sqrt[3]{\frac{3V}{4\pi}}$$

Solve the equation for V.

[6–6]

Find the solution set of each equation. Identify extraneous solutions, if they exist.

35. $x^4 - 5x^2 - 14 = 0$

36. $(y + 1)^2 - 6(y + 1) + 5 = 0$

37. $q + 9\sqrt{q} + 8 = 0$

38. $3p - 5p^{1/2} - 2 = 0$

39. $2x^{2/3} + x^{1/3} - 6 = 0$

40. $z^{-2} = 10z^{-1} + 11$

41. $5y^{-4} - 8y^{-2} = 4$

[6–7]

Find the solution set and graph each inequality. Write the answer in set-builder and interval notation.

42. $(x - 3)(x + 7) > 0$

43. $(2x - 1)(3x + 4) \leq 0$

44. $2y^2 - 3y \geq 0$

45. $9z^2 - 4 < 0$

46. $m^2 - 7 > 6m$

47. $4p^2 - 5p < 6$

48. $5x^2 \geq 45$

49. $y^2 - 14y \leq 32$

50. $(y - 2)(y + 4)(y - 5) < 0$

51. $\dfrac{m + 3}{m - 1} \geq 0$

52. $\dfrac{x - 3}{x + 7} \leq 2$

Chapter 6 cumulative test

Perform the indicated operations and simplify. Assume all variables are nonzero real numbers. Leave all answers with positive exponents.

[1–2] 1. $\dfrac{(-18)(-4)}{-6}$

[1–4] 2. $16 - 8[3 - 4(12 - 8) + 14] - 6$

[1–4] 3. $56 - 28 \div 7 - 5 + 3^2$

[1–6] 4. $(4xy^2 - 5xy + 2x^2y) - (-3xy^2 + xy - 4x^2y)$

[3–2] 5. $(3x + 2y)^2$

[3–2] 6. $(4y + 1)(4y - 1)$

[3–2] 7. $(x - 2)(9x^2 + 2x + 4)$

[3–1] 8. $(5xy^2)^3(-3xy)^3$

[3–3] 9. $\left(\dfrac{a^2b^0}{a^{-3}}\right)^4$

[3–3] 10. $\dfrac{-24a^{-3}b^2}{12a^2b^{-3}}$

[1–6] 11. Given $P(x) = 3x^2 - 2x + 1$, find (a) $P(-1)$, (b) $P(0)$, (c) $P(4)$.

Reduce the following rational expressions to lowest terms. Assume all denominators are nonzero.

[4–1] 12. $\dfrac{12a - 12b}{b - a}$

[4–1] 13. $\dfrac{6a^2 - 6}{3a^2 - 15a + 12}$

[4–1] 14. $\dfrac{3b^2 - 12}{4b^2 - 16}$

Perform the indicated operations and simplify. All denominators are nonzero.

[4–2] 15. $\dfrac{x + 2}{x - 1} \cdot \dfrac{x^2 - 1}{x^2 - x - 6}$

[4–2] 16. $\dfrac{x^2 - 2x + 1}{x^2 - 49} \div \dfrac{x^2 - 7x + 6}{2x^2 - 15x + 7}$

[4–3] 17. $\dfrac{15}{p^2 - 7p - 18} - \dfrac{3}{p^2 - 4}$

[4–3] 18. $\dfrac{4a + 1}{a - 6} + \dfrac{3a - 4}{6 - a}$

Find the solution set of the following equations and inequalities.

[2–4] 19. $|4x + 7| = 5$

[2–6] 20. $|2x - 5| \le 5$

[2–6] 21. $|7 - 2x| > 4$

[2–1] 22. $4(2x + 1) - 3(x - 1) = 4x$

[2–5] 23. $4x - 3 \le 5(x + 6)$

[4–7] 24. $\dfrac{3}{x} - \dfrac{4}{x} = \dfrac{6}{10}$

[2–2] 25. Solve $P = 2\ell + 2w$ for w.

[4–4] 26. Simplify the complex fraction $\dfrac{5 - \dfrac{6}{3y}}{2 + \dfrac{5}{2y}}$.

Perform the indicated operations and simplify.

[5–6] 27. $\sqrt{3}(6 - \sqrt{3})$

[5–5] 28. $3\sqrt{75} + \sqrt{27} - \sqrt{12}$

[5–5] 29. $\sqrt[3]{81} - 3\sqrt[3]{24}$

[5–7] 30. $(3 - 2i)(3 + 2i)$

[5–6] 31. $(2\sqrt{5} - 3\sqrt{3})^2$

Rationalize the denominator.

[5–6] 32. $\sqrt{\dfrac{16}{5}}$

[5–6] 33. $\dfrac{\sqrt{2} - \sqrt{3}}{\sqrt{2} + \sqrt{3}}$

[5–7] 34. $\dfrac{i}{2 - 5i}$

Find the solution sets of the following equations and inequalities.

[6–1] 35. $x^2 - 15x + 14 = 0$

[6–3] 36. $5y^2 - y + 3 = 0$

[6–3] 37. $3y^2 + 1 = y - 2$

[6–7] 38. $z^2 - 2z \le 3$

[6–6] 39. $p^4 - 5p^2 - 50 = 0$

[6–7] 40. $\dfrac{2y - 3}{y + 7} < 1$

[4–6] 41. Divide $4x^5 - 3x^2 + x^2 - x + 1$ by $x + 1$ using synthetic division.

Chapter 5 review

1. 6 **2.** $\dfrac{1}{8}$ **3.** 9 **4.** $a^{11/12}$ **5.** $c^{1/4}$ **6.** $9x^2$ **7.** b^2

8. $a^{1/6}$ **9.** $4x^8y^4$ **10.** $x^{19/6}$ **11.** $a^{7/6}$ **12.** $2\sqrt{3}$

13. $5\sqrt{6}$ **14.** $x\sqrt[5]{x^2}$ **15.** $2ab\sqrt[3]{3b}$ **16.** $a\sqrt{a}$

17. $\sqrt[3]{2ab^2}$ **18.** 8 in. **19.** $\dfrac{7}{8}$ **20.** $\dfrac{4\sqrt{3}}{9}$ **21.** $\dfrac{2x\sqrt[3]{2y^2}}{z^2}$

22. $\dfrac{\sqrt{2}}{4}$ **23.** $3\sqrt{2}$ **24.** $\dfrac{2\sqrt[3]{5}}{5}$ **25.** $\dfrac{x\sqrt{y}}{y}$ **26.** $\dfrac{\sqrt[3]{a^3b^2}}{b}$

27. $\sqrt[5]{x^3}$ **28.** $\dfrac{\sqrt[3]{ab^2c}}{bc}$ **29.** $\dfrac{\sqrt[4]{a^3b^2}}{b}$ **30.** $\dfrac{x\sqrt{y}}{y}$ **31.** $8\sqrt{3}$

32. $8\sqrt{2}$ **33.** $29\sqrt{2a}$ **34.** $3x^2\sqrt{xy}$ **35.** $\dfrac{5\sqrt{6}-2\sqrt{3}}{6}$

36. $\dfrac{2\sqrt{ab}-b\sqrt{a}}{ab}$ **37.** $2\sqrt{3}-2\sqrt{5}$ **38.** $2a\sqrt{b}+4a$

39. $30-10\sqrt{5}$ **40.** 3 **41.** $4a-9b$ **42.** $9x+6y\sqrt{x}+y^2$

43. $\dfrac{\sqrt{6}-2}{2}$ **44.** $4-\sqrt{6}$ **45.** $\sqrt{2}+1$ **46.** $\dfrac{a^2b\sqrt{a}+ab\sqrt{ab}}{a^2-b}$

47. $7i$ **48.** $2i\sqrt{7}$ **49.** -4 **50.** -7 **51.** -6 **52.** -3

53. $-\sqrt{6}$ **54.** i **55.** $7+7i$ **56.** $-1-11i$ **57.** $18-i$

58. $-21+20i$ **59.** $4-3i$ **60.** $-2-\dfrac{7}{3}i$ **61.** $\dfrac{7}{5}-\dfrac{6}{5}i$

62. $\dfrac{69}{58}+\dfrac{13}{58}i$

Chapter 5 cumulative test

1. $(a-8)(a+1)$ **2.** $x(4x-3)$ **3.** $9(x-2)(x+2)$

4. $(2x+3)(x+4)$ **5.** $(3a+4)(a-5)$

6. $(3x+4)(2x+3)$ **7.** (a) 28, (b) -8 **8.** $\left\{\dfrac{2}{5}\right\}$

9. $\left\{x\,|\,x>-\dfrac{11}{3}\right\}$ **10.** $x=-6y$ **11.** $\left\{-1,\dfrac{5}{3}\right\}$

12. $\left\{x\,|\,x<-\dfrac{11}{2}\text{ or }x>\dfrac{5}{2}\right\}$ **13.** $\left\{-\dfrac{13}{5}\right\}$

14. $\left\{x\,|\,-\dfrac{3}{2}\le x\le 2\right\}$ **15.** $2a^2b\sqrt[5]{2b^2}$ **16.** $10-5i$

17. $4\sqrt{3}$ **18.** $a^{7/12}$ **19.** $2ab^2\sqrt[3]{a}$ **20.** $8a^9b^{12}c^3$ **21.** $3i\sqrt{2}$

22. $\sqrt{10}-\sqrt{6}$ **23.** $-\dfrac{7}{13}-\dfrac{9}{13}i$ **24.** $2a^3$ **25.** $\dfrac{x^3y^2}{3}$

26. $\dfrac{\sqrt[3]{2a^2b^2c}}{2bc}$ **27.** 7 inches **28.** 400 kg of 80% copper,

600 kg of 50% copper **29.** 375 meters per second

30. 1,166.4 meters per second

Chapter 6

Exercise 6–1

Answers to odd-numbered problems

1. $\{3,-4\}$ **3.** $\left\{\dfrac{1}{3},-\dfrac{5}{2}\right\}$ **5.** $\{2,3\}$ **7.** $\{5\}$ **9.** $\{-3,8\}$

11. $\{0,1\}$ **13.** $\{-3,3\}$ **15.** $\left\{-\dfrac{1}{2},2\right\}$ **17.** $\left\{-2,\dfrac{3}{4}\right\}$

19. $\{2\}$ **21.** $\{-8,1\}$ **23.** $\{-5,1\}$ **25.** $\left\{-\dfrac{1}{3},2\right\}$

27. $\left\{-\dfrac{5}{2},3\right\}$ **29.** $\{-6,1\}$ **31.** $\{-11,11\}$ **33.** $\{-7,7\}$

35. $\{-4\sqrt{2},4\sqrt{2}\}$ **37.** $\{-6\sqrt{2},6\sqrt{2}\}$ **39.** $\{-2\sqrt{2},2\sqrt{2}\}$

41. $\{-5\sqrt{2},5\sqrt{2}\}$ **43.** $\{-1,-13\}$ **45.** $\{12+11i,12-11i\}$

47. $\{-10+4\sqrt{3},-10-4\sqrt{3}\}$ **49.** $\left\{2,-\dfrac{5}{2}\right\}$

51. $\left\{\dfrac{3+2i\sqrt{21}}{10},\dfrac{3-2i\sqrt{21}}{10}\right\}$ **53.** $\{-8-b,-8+b\}$

55. $x=-2b,12b$ **57.** $x=-\dfrac{7a}{4},2a$ **59.** $x=y$

61. $x=-\dfrac{4y}{3},\dfrac{y}{2}$ **63.** (a) $t=4$ sec, (b) $t=2$ sec

65. $t=1$ sec **67.** $n=7$ **69.** 7 meters **71.** 8,15,17

73. 4,6; $-6,-4$ **75.** 7,9; $-7,-9$

Solutions to trial exercise problems

13. $-3y^2+27=0$
$-3(y^2-9)=0$
$-3(y+3)(y-3)=0$
$y=-3$ when $y+3=0$, $y=3$
when $y-3=0$
The solution set is $\{-3,3\}$.

21. $\dfrac{x}{2}+\dfrac{7}{2}=\dfrac{4}{x}$
Multiply each member by the LCM, $2x$.
$2x\cdot\dfrac{x}{2}+2x\cdot\dfrac{7}{2}=2x\cdot\dfrac{4}{x}$
$x^2+7x=8$
$x^2+7x-8=0$
$(x+8)(x-1)=0$
$x=-8$ when $x+8=0$ and
$x=1$ when $x-1=0$
The solution set is $\{-8,1\}$.

23. $(y+6)(y-2)=-7$
$y^2+4y-12=-7$
$y^2+4y-5=0$
$(y+5)(y-1)=0$
$y=-5$ when $y+5=0$
and $y=1$ when $y-1=0$
The solution set is $\{-5,1\}$

44. $(x-9)^2=-144$
$x-9=\sqrt{-144}=12i$ or $x-9=-\sqrt{-144}=-12i$
Then $x=9+12i$ or $x=9-12i$
The solution set is $\{9+12i,9-12i\}$

52. $(x-7)^2=a^2, a>0$
$x-7=\sqrt{a^2}=a$ or $x-7=-\sqrt{a^2}=-a$
Then $x=7+a$ or $x=7-a$
$\{7-a,7+a\}$

56. $3x^2-13xy+4y^2=0$
$(3x-y)(x-4y)=0$
$x=\dfrac{y}{3}$ when $3x-y=0$ and
$x=4y$ when $x-4y=0$, so
$x=\dfrac{y}{3}$ or $x=4y$.

62. a. $P=100I-5I^2$
$420=100I-5I^2$
$5I^2-100I+420=0$
$5(I^2-20I+84)=0$
$5(I-6)(I-14)=0$
$I=6$ when $I-6=0$ and $I=14$ when $I-14=0$
So $P=420$ when $I=6$ amperes or $I=14$ amperes.

Review exercises

1. $\dfrac{y^4}{x^4}$ 2. $\dfrac{x^6}{16y^4}$ 3. $4x^2 - 12x + 9$ 4. $x^2 + 14x + 49$

5. $\{x|-1 \le x < 3\} = [-1,3)$ 6. $\left\{\dfrac{1}{4},3\right\}$

Exercise 6–2

Answers to odd-numbered problems

1. $x^2 + 4x + 4 = (x + 2)^2$ 3. $y^2 - 18y + 81 = (y - 9)^2$

5. $p^2 + 2p + 1 = (p + 1)^2$ 7. $x^2 + 3x + \dfrac{9}{4} = \left(x + \dfrac{3}{2}\right)^2$

9. $w^2 - 11w + \dfrac{121}{4} = \left(w - \dfrac{11}{2}\right)^2$

11. $x^2 + 13x + \dfrac{169}{4} = \left(x + \dfrac{13}{2}\right)^2$ 13. $\{-11,-1\}$

15. $\{1,10\}$ 17. $\{-4 + 3i,-4 - 3i\}$ 19. $\{0,8\}$

21. $\left\{\dfrac{-1 + \sqrt{13}}{2},\dfrac{-1 - \sqrt{13}}{2}\right\}$ 23. $\left\{\dfrac{3 - \sqrt{17}}{2},\dfrac{3 + \sqrt{17}}{2}\right\}$

25. $\left\{-2,\dfrac{1}{2}\right\}$ 27. $\{2 - \sqrt{3},2 + \sqrt{3}\}$

29. $\left\{-\dfrac{1}{2},\dfrac{3}{2}\right\}$ 31. $\left\{\dfrac{5 - \sqrt{5}}{10},\dfrac{5 + \sqrt{5}}{10}\right\}$

33. $\left\{\dfrac{1 + \sqrt{29}}{2},\dfrac{1 - \sqrt{29}}{2}\right\}$ 35. $\left\{\dfrac{1}{4},-\dfrac{3}{2}\right\}$

37. $\left\{\dfrac{1 - \sqrt{3}}{2},\dfrac{1 + \sqrt{3}}{2}\right\}$ 39. $\left\{\dfrac{3 + \sqrt{41}}{4},\dfrac{3 - \sqrt{41}}{4}\right\}$

41. $\left\{\dfrac{2 - \sqrt{22}}{3},\dfrac{2 + \sqrt{22}}{3}\right\}$ 43. $\left\{\dfrac{2 - \sqrt{10}}{2},\dfrac{2 + \sqrt{10}}{2}\right\}$

45. $\left\{\dfrac{5}{2},-1\right\}$ 47. $\left\{-\dfrac{3}{5},1\right\}$

49. $t = \dfrac{-9 + \sqrt{6,481}}{32}$ sec ≈ 2.23 sec

51. $p = -16 + 6\sqrt{21}$ ¢ ≈ 12¢   53. $h = 25$ or $h = 1$

55. $\dfrac{1 + \sqrt{65}}{8};\dfrac{1 - \sqrt{65}}{8}$ 57. $\sqrt{\dfrac{235}{10\pi}} = \dfrac{\sqrt{94\pi}}{2\pi}$

Solutions to trial exercise problems

7. $\left[\dfrac{1}{2}(3)\right]^2 = \left(\dfrac{3}{2}\right)^2 = \dfrac{9}{4}$

$x^2 + 3x + \dfrac{9}{4} = \left(x + \dfrac{3}{2}\right)^2$

20. $x^2 + 4x = 0$
$x^2 + 4x + 4 = 4$
$(x + 2)^2 = 4$
$x + 2 = \pm 2$
$x = -2 \pm 2$
$x = 0$ or $x = -4$
The solution set is $\{0,-4\}$.

21. $y^2 = 3 - y$
$y^2 + y = 3$
$y^2 + y + \dfrac{1}{4} = 3 + \dfrac{1}{4}$
$\left(y + \dfrac{1}{2}\right)^2 = \dfrac{13}{4}$
$y + \dfrac{1}{2} = \pm \dfrac{\sqrt{13}}{2}$
$y = -\dfrac{1}{2} \pm \dfrac{\sqrt{13}}{2} = \dfrac{-1 \pm \sqrt{13}}{2}$
The solution set is $\left\{\dfrac{-1 + \sqrt{13}}{2},\dfrac{-1 - \sqrt{13}}{2}\right\}$.

33. $(x + 2)(x - 3) = 1$
$x^2 - x - 6 = 1$
$x^2 - x = 7$
$x^2 - x + \dfrac{1}{4} = 7 + \dfrac{1}{4}$
$\left(x - \dfrac{1}{2}\right)^2 = \dfrac{29}{4}$
$x - \dfrac{1}{2} = \pm \dfrac{\sqrt{29}}{2}$
$x = \dfrac{1}{2} \pm \dfrac{\sqrt{29}}{2} = \dfrac{1 \pm \sqrt{29}}{2}$
The solution set is $\left\{\dfrac{1 + \sqrt{29}}{2},\dfrac{1 - \sqrt{29}}{2}\right\}$.

39. $\dfrac{1}{2}x^2 - \dfrac{3}{4}x = 1$ Multiply by the LCM, 4.

$2x^2 - 3x = 4$ Multiply by $\dfrac{1}{2}$.

$x^2 - \dfrac{3}{2}x = 2$

$x^2 - \dfrac{3}{2}x + \dfrac{9}{16} = 2 + \dfrac{9}{16}$

$\left(x - \dfrac{3}{4}\right)^2 = \dfrac{41}{16}$

$x - \dfrac{3}{4} = \pm \dfrac{\sqrt{41}}{4}$

$x = \dfrac{3}{4} \pm \dfrac{\sqrt{41}}{4} = \dfrac{3 \pm \sqrt{41}}{4}$

The solution set is $\left\{\dfrac{3 + \sqrt{41}}{4},\dfrac{3 - \sqrt{41}}{4}\right\}$.

45. $\dfrac{5}{x} - 2x + 3 = 0$

$5 - 2x^2 + 3x = 0$ Multiply by the LCM, x.
$2x^2 - 3x - 5 = 0$ Multiply each member by -1.
$2x^2 - 3x = 5$

$x^2 - \dfrac{3}{2}x = \dfrac{5}{2}$

$x^2 - \dfrac{3}{2}x + \dfrac{9}{16} = \dfrac{5}{2} + \dfrac{9}{16}$

$\left(x - \dfrac{3}{4}\right)^2 = \dfrac{49}{16}$

$x - \dfrac{3}{4} = \pm \dfrac{7}{4}$

$x = \dfrac{3}{4} \pm \dfrac{7}{4} = \dfrac{3 \pm 7}{4}$

The solution set is $\left\{\dfrac{5}{2},-1\right\}$.

49. $s = 100$, so $100 = 9t + 16t^2$.

$$16t^2 + 9t - 100 = 0$$

$$t^2 + \frac{9}{16}t - \frac{100}{16} = 0$$

$$t^2 + \frac{9}{16}t = \frac{25}{4}$$

$$t^2 + \frac{9}{16}t + \frac{81}{1,024} = \frac{25}{4} + \frac{81}{1,024}$$

$$\left(t + \frac{9}{32}\right)^2 = \frac{6,400 + 81}{1,024} = \frac{6,481}{1,024}$$

$$t + \frac{9}{32} = \pm\frac{\sqrt{6,481}}{32}$$

$$t = -\frac{9}{32} \pm \frac{\sqrt{6,481}}{32} = \frac{-9 \pm \sqrt{6,481}}{32}$$

Therefore $t = \dfrac{-9 + \sqrt{6,481}}{32}$ sec ≈ 2.23 sec. (We discard the other value of t since $t > 0$.)

Review exercises

1. $2\sqrt{3} - \sqrt{15}$ **2.** 1   **3.** $-\sqrt{6}$ **4.** $27 - 18\sqrt{2}$

5. $4x - 2 + \dfrac{1}{x}$ **6.** $4x + 9 + \dfrac{25}{x - 3}$ **7.** 1   **8.** 61

9. -12

Exercise 6–3

Answers to odd-numbered problems

1. $\left\{\dfrac{-5 + i\sqrt{3}}{2}, \dfrac{-5 - i\sqrt{3}}{2}\right\}$ **3.** $\left\{\dfrac{3 - i\sqrt{11}}{2}, \dfrac{3 + i\sqrt{11}}{2}\right\}$

5. $\left\{2, \dfrac{3}{2}\right\}$ **7.** $\left\{\dfrac{2 + \sqrt{3}}{2}, \dfrac{2 - \sqrt{3}}{2}\right\}$

9. $\left\{\dfrac{1 - 2i\sqrt{5}}{3}, \dfrac{1 + 2i\sqrt{5}}{3}\right\}$ **11.** $\{-8, 2\}$ **13.** $\{7\}$

15. $\left\{\dfrac{-2\sqrt{15}}{3}, \dfrac{2\sqrt{15}}{3}\right\}$ **17.** $\left\{0, \dfrac{2}{5}\right\}$ **19.** $\left\{\dfrac{2}{3}\right\}$

21. $\left\{\dfrac{2 - i}{3}, \dfrac{2 + i}{3}\right\}$ **23.** $\left\{\dfrac{-1 - 2\sqrt{3}}{3}, \dfrac{-1 + 2\sqrt{3}}{3}\right\}$

25. $\left\{\dfrac{1 - i\sqrt{55}}{4}, \dfrac{1 + i\sqrt{55}}{4}\right\}$ **27.** $\left\{\dfrac{1 + \sqrt{113}}{8}, \dfrac{1 - \sqrt{113}}{8}\right\}$

29. $\left\{\dfrac{3 + i\sqrt{3}}{4}, \dfrac{3 - i\sqrt{3}}{4}\right\}$ **31.** $\left\{-2, \dfrac{8}{3}\right\}$

33. $\left\{\dfrac{11 + \sqrt{145}}{6}, \dfrac{11 - \sqrt{145}}{6}\right\}$ **35.** $\left\{\dfrac{3 + \sqrt{21}}{2}, \dfrac{3 - \sqrt{21}}{2}\right\}$

37. $\left\{-4, \dfrac{2}{3}\right\}$ **39.** $x = -3y, x = 6y$

41. $x = \dfrac{-1 \pm \sqrt{1 + 12y}}{4}$ **43.** $x = 2a \pm \sqrt{4a^2 - 3a}$

45. two, irrational   **47.** two, complex   **49.** one, rational
51. two, rational   **53.** two, irrational   **55.** two, irrational
57. two, irrational   **59.** two, irrational   **61.** two, irrational

63. $c = 15, a = 9, b = 12$ **65.** (a) $\dfrac{-3 + \sqrt{73}}{4}$ sec,

(b) $\dfrac{-9 + \sqrt{401}}{8}$ sec   **67.** $r = 9.5\%$

Solutions to trial exercise problems

19. $9x^2 - 12x + 4 = 0$

$a = 9, b = -12, c = 4$

$$x = \frac{-(-12) \pm \sqrt{(-12)^2 - 4(9)(4)}}{2(9)}$$

$$= \frac{12 \pm \sqrt{144 - 144}}{18} = \frac{12 \pm \sqrt{0}}{18} = \frac{12}{18} = \frac{2}{3}$$

The solution set is $\left\{\dfrac{2}{3}\right\}$.

26. $3x - \dfrac{2}{x} + 5 = 0$

$3x^2 - 2 + 5x = 0$

$3x^2 + 5x - 2 = 0$

$a = 3, b = 5, c = -2$

$$x = \frac{-5 \pm \sqrt{5^2 - 4(3)(-2)}}{2(3)}$$

$$x = \frac{-5 \pm \sqrt{25 + 24}}{6}$$

$$= \frac{-5 \pm \sqrt{49}}{6} = \frac{-5 \pm 7}{6}$$

$$x = \frac{-5 + 7}{6} = \frac{2}{6} = \frac{1}{3} \text{ or } x = \frac{-5 - 7}{6} = \frac{-12}{6} = -2$$

The solution set is $\left\{-2, \dfrac{1}{3}\right\}$.

27. $2y^2 - \dfrac{7}{2} = \dfrac{y}{2}$

$4y^2 - 7 = y$

$4y^2 - y - 7 = 0$

$a = 4, b = -1, c = -7$

$$y = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(4)(-7)}}{2(4)}$$

$$= \frac{1 \pm \sqrt{1 + 112}}{8} = \frac{1 \pm \sqrt{113}}{8}$$

The solution set is $\left\{\dfrac{1 + \sqrt{113}}{8}, \dfrac{1 - \sqrt{113}}{8}\right\}$.

32. $\dfrac{1}{x + 2} + \dfrac{1}{x - 3} - 2 = 0$

Multiply by the LCM, $(x + 2)(x - 3)$.

$(x - 3) + (x + 2) - 2(x + 2)(x - 3) = 0$

$x - 3 + x + 2 - 2(x^2 - x - 6) = 0$

$2x - 1 - 2x^2 + 2x + 12 = 0$

$-2x^2 + 4x + 11 = 0$

Multiply by -1 to get $2x^2 - 4x - 11 = 0$.

$a = 2, b = -4, \text{ and } c = -11$

$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(2)(-11)}}{2(2)}$$

$$= \frac{4 \pm \sqrt{16 + 88}}{4} = \frac{4 \pm \sqrt{104}}{4}$$

$$= \frac{4 \pm 2\sqrt{26}}{4} = \frac{2(2 \pm \sqrt{26})}{4} = \frac{2 \pm \sqrt{26}}{2}$$

The solution set is $\left\{\dfrac{2 + \sqrt{26}}{2}, \dfrac{2 - \sqrt{26}}{2}\right\}$.

35. $(z - 3)(z + 2) = 2z - 3$

$z^2 - z - 6 = 2z - 3$

$z^2 - 3z - 3 = 0$

$a = 1, b = -3, c = -3$

$$z = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(1)(-3)}}{2(1)}$$

$$= \frac{3 \pm \sqrt{9 + 12}}{2} = \frac{3 \pm \sqrt{21}}{2}$$

The solution set is $\left\{\dfrac{3 + \sqrt{21}}{2}, \dfrac{3 - \sqrt{21}}{2}\right\}$.

41. $4x^2 + 2x - 3y = 0$

$a = 4, b = 2, c = -3y$

$$x = \frac{-2 \pm \sqrt{(2)^2 - 4(4)(-3y)}}{2(4)}$$

$$= \frac{-2 \pm \sqrt{4 + 48y}}{8} = \frac{-2 \pm \sqrt{4(1 + 12y)}}{8}$$

$$= \frac{-2 \pm 2\sqrt{1 + 12y}}{8}$$

$$= \frac{-1 \pm \sqrt{1 + 12y}}{4}$$

$$x = \frac{-1 \pm \sqrt{1 + 12y}}{4}$$

62. (b) $60 = \dfrac{1}{2}(32)t^2$

$60 = 16t^2$

$16t^2 - 60 = 0$

$a = 16, b = 0, c = -60$

$$t = \frac{0 \pm \sqrt{0^2 - 4(16)(-60)}}{2(16)} = \frac{\pm \sqrt{3,840}}{32}$$

$$= \frac{\pm 16\sqrt{15}}{32} = \frac{\pm \sqrt{15}}{2}$$

Then $t = \dfrac{\sqrt{15}}{2} \approx 1.9$ sec.

$\left(\text{Discard } t = \dfrac{-\sqrt{15}}{2} \text{ since } t > 0.\right)$

Review exercises

1. $-11\sqrt{7}$ **2.** $\dfrac{3}{5}$ **3.** $14 - 6\sqrt{5}$ **4.** $22 + 7i$

5. $-2 - 2i\sqrt{3}$ **6.** $-23x^{10}$

Exercise 6–4

Answers to odd-numbered problems

1. $t = 1$ sec   **3.** $t = 3$ sec   **5.** $t = 6$ sec   **7.** $t = \dfrac{-8 + \sqrt{619}}{4}$

≈ 4.2 sec   **9.** $t = 0$ sec and $t = 1.5$ sec   **11.** 30 amperes
13. 30 sides   **15.** \$3.00   **17.** (a) 15 pens, (b) 15 pens
19. $-1 + \sqrt{51} \approx 6$ units   **21.** 120 cakes   **23.** $20 \pm 8\sqrt{5}$
≈ 37.9 or 2.1   **25.** $-15 + 15\sqrt{127}$ ft ≈ 154 ft by $15 + 15\sqrt{127}$
ft ≈ 184 ft   **27.** 5 ft   **29.** 13 in.   **31.** 50 yd by 100 yd
33. $D = 4$ ft   **35.** $7 + 4\sqrt{7} \approx 17.6$ in.   **37.** 25 in.
39. Lisa, 1 hr 5 min (65 min); Debbie, 1 hr 44 min (104 min)
41. $-1 + \sqrt{17}$ hr ≈ 3.1 hr

Solutions to trial exercise problems

4. $h = 80$ and $v_0 = 96$, so $80 = 96t - 16t^2$, then

$16t^2 - 96t + 80 = 0$

$16(t^2 - 6t + 5) = 0$

$16(t - 5)(t - 1) = 0$

The object reaches $h = 80$ feet at $t = 1$ sec.

Note: The object is at $h = 80$ feet again at $t = 5$ sec, on its way down to earth.

20. Using $20,000 = \dfrac{1}{100}n^2 - 20n,$

$2,000,000 = n^2 - 2,000n$

$n^2 - 2,000n - 2,000,000 = 0$

$$n = \frac{-(-2,000) \pm \sqrt{(-2,000)^2 - 4(1)(-2,000,000)}}{2(1)}$$

$$= \frac{2,000 \pm \sqrt{4,000,000 + 8,000,000}}{2}$$

$$= \frac{2,000 \pm \sqrt{12,000,000}}{2} = \frac{2,000 \pm 2,000\sqrt{3}}{2}$$

Then $n = \dfrac{2,000 + 2,000\sqrt{3}}{2} = 1,000 + 1,000\sqrt{3}$

$\approx 2,732$ $(n > 0)$.

Thus about 2,732 units must be produced to make a \$20,000 profit.

27. Let $s =$ the length of the side of the square.

Using $s^2 + s^2 = (\sqrt{50})^2$

$2s^2 = 50$

so $2s^2 - 50 = 0$

$2(s^2 - 25) = 0$

$2(s - 5)(s + 5) = 0$, so $s = 5$ or $s = -5$.
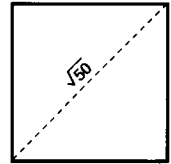
Then $s = 5$ feet. (*Note:* Reject $s = -5$ since the length of the side cannot be negative.)

30. Let $x =$ the amount the dimensions are increased. Since the area of the original rectangle is 18 cm², and the new rectangle has area $3 \cdot 18 = 54$ cm², then

$(x + 6)(x + 3) = 54$

$x^2 + 9x + 18 = 54$

$x^2 + 9x - 36 = 0$

$(x + 12)(x - 3) = 0$, then $x = -12$ or $x = 3$.

Reject -12, so $x = 3$ cm and the new dimensions are $3 + 6 = 9$ cm long and $3 + 3 = 6$ cm wide.

39. Let x = the time in minutes for Lisa to do the job. Then
$x + 39$ = the time in minutes for Debbie to do the job.

Then $\dfrac{1}{x} + \dfrac{1}{x + 39} = \dfrac{1}{40}$

$40(x + 39) + 40x = x(x + 39)$

$40x + 1{,}560 + 40x = x^2 + 39x$

$x^2 - 41x - 1{,}560 = 0$

$x = \dfrac{-(-41) \pm \sqrt{(-41)^2 - 4(1)(-1{,}560)}}{2(1)}$

$= \dfrac{41 \pm \sqrt{1{,}681 + 6{,}240}}{2} = \dfrac{41 \pm \sqrt{7{,}921}}{2} = \dfrac{41 \pm 89}{2}$

Thus $x = \dfrac{41 + 89}{2} = \dfrac{130}{2} = 65$ or $x = \dfrac{41 - 89}{2} = \dfrac{-48}{2}$

$= -24$ (reject this answer).

Therefore Lisa can do the job in 65 minutes and Debbie can do the job in $65 + 39 = 104$ minutes.

Review exercises

1. $28 - 16\sqrt{3}$ **2.** $\left\{-\dfrac{2}{7}\right\}$ **3.** $\{-3,8\}$ **4.** 8   **5.** $x^{11/6}$

6. $x^{1/6}$ **7.** $\dfrac{\sqrt{6}}{3}$

Exercise 6–5

Answers to odd-numbered problems

1. $\{81\}$ **3.** $\emptyset$ **5.** $\{53\}$ **7.** $\{24\}$ **9.** $\{1\}$ **11.** $\{1\}$ **13.** $\{1\}$
15. $\{9\}$; -4 is extraneous   **17.** $\{6\}$; -3 is extraneous
19. $\{-\sqrt{2},\sqrt{2}\}$ **21.** $\{-9,1\}$ **23.** $\{-3,3\}$ **25.** $\{5\}$; -2 is

extraneous   **27.** $\left\{\dfrac{1}{2},2\right\}$ **29.** $\{8\}$; 2 is extraneous   **31.** $\{1,4\}$

33. $\{5\}$; 2 is extraneous   **35.** $\left\{\dfrac{5}{2}\right\}$; 2 is extraneous   **37.** $\{0,3\}$

39. $\{3,11\}$ **41.** $\{0,8\}$ **43.** $\emptyset$; $\dfrac{3}{4}$ is extraneous

45. $\left\{-\dfrac{20}{9},-4\right\}$ **47.** $\{34\}$ **49.** $\{-2,8\}$

51. $\left\{-\dfrac{5}{4}\right\}$ **53.** $A = 4\pi r^2 h$ **55.** $A = \pi r^2 + \pi R^2$

57. $A = \dfrac{\pi D^3}{6}$ **59.** 6 ft   **61.** 75 ft   **63.** -27

Solutions to trial exercise problems

14. $\sqrt{p}\,\sqrt{p - 8} = 3$
Squaring, $p(p - 8) = 9$
$p^2 - 8p = 9$
$p^2 - 8p - 9 = 0$
$(p - 9)(p + 1) = 0$
$p = 9$ or $p = -1$
The solution set is $\{9\}$.
-1 is an extraneous solution.

34. $\sqrt{3x + 10} - 3x = 4$
$\sqrt{3x + 10} = 3x + 4$
Square both sides.
$3x + 10 = 9x^2 + 24x + 16$
$9x^2 + 21x + 6 = 0$
$3(3x^2 + 7x + 2) = 0$
$3(3x + 1)(x + 2) = 0$
$x = -\dfrac{1}{3}$ or $x = -2$

The solution set is $\left\{-\dfrac{1}{3}\right\}$.
-2 is an extraneous solution.

37. $\sqrt{5x + 1} - 1 = \sqrt{3x}$
$\sqrt{5x + 1} = 1 + \sqrt{3x}$
Squaring, $5x + 1 = 1 + 2\sqrt{3x} + 3x$
$2x = 2\sqrt{3x}$
$x = \sqrt{3x}$
$x^2 = 3x$
$x^2 - 3x = 0$
$x(x - 3) = 0$
$x = 0$ or $x = 3$
The solution set is $\{0,3\}$.

42. $(2y + 3)^{1/2} - (4y - 1)^{1/2} = 0$
$\sqrt{2y + 3} = \sqrt{4y - 1}$
Squaring, $2y + 3 = 4y - 1$
$4 = 2y$
$y = 2$
The solution set is $\{2\}$.

47. $\sqrt[3]{x - 7} = 3$
Cubing, $x - 7 = 27$
$x = 34$
The solution set is $\{34\}$.

55. $r = \sqrt{\dfrac{A}{\pi} - R^2}$

$r^2 = \dfrac{A}{\pi} - R^2$

$\pi r^2 = A - \pi R^2$

$A = \pi r^2 + \pi R^2$

62. Let x = the number.
Then $\sqrt{x} = 3i$ (Square each member.)
$x = 9(-1)$
$x = -9$.

Review exercises

1. $\{-8,2\}$ **2.** $\left\{\dfrac{-1 - \sqrt{7}}{3}, \dfrac{-1 + \sqrt{7}}{3}\right\}$ **3.** $\{y|y \geq 4\} = [4,\infty)$

4. $\{x|-2 < x \leq 1\} = (-2,1]$ **5.** 18   **6.** $21° \leq t \leq 62°$

Exercise 6–6

Answers to odd-numbered problems

1. $\{-\sqrt{5},\sqrt{5},-1,1\}$ **3.** $\left\{-\dfrac{\sqrt{6}}{3},-i,i,\dfrac{\sqrt{6}}{3}\right\}$ **5.** $\{2 \pm 2i\sqrt{2}, 2 \pm i\}$

7. $\{-2,1,4\}$ **9.** $\{81\}$; 9 is extraneous **11.** $\emptyset$ **13.** $\{1,16\}$

15. $\{2\sqrt{13},-2\sqrt{13}\}$; $-\sqrt{19}$ and $\sqrt{19}$ are extraneous

17. $\{10\}$; 7 is extraneous **19.** $\{-125,8\}$ **21.** $\left\{-\dfrac{1}{125},-8\right\}$

23. $\{1\}$ **25.** $\left\{\dfrac{1}{4},-\dfrac{1}{3}\right\}$ **27.** $\left\{\dfrac{1}{4},2\right\}$ **29.** $\left\{-2,-\dfrac{1}{2},\dfrac{1}{2},2\right\}$

31. $\{-5,-2,-1,2\}$ **33.** $\{4\}$; 36 is extraneous

Solutions to trial exercise problems
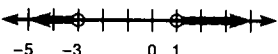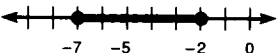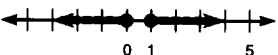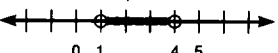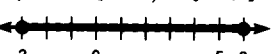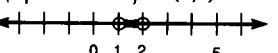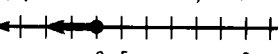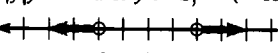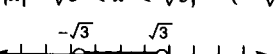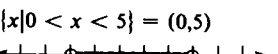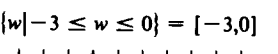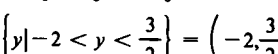
5. $(x-2)^4 + 9(x-2)^2 + 8 = 0$
Let $u = (x-2)^2$
$u^2 + 9u + 8 = 0$
$(u+8)(u+1) = 0$
$u = -8 \text{ or } u = -1$
Substitute $(x-2)^2$ for u.
$(x-2)^2 = -8 \text{ or } (x-2)^2 = -1$
$x - 2 = \pm\sqrt{-8} \text{ or } x - 2 = \pm\sqrt{-1}$
$x - 2 = \pm2i\sqrt{2} \text{ or } x - 2 = \pm i$
$x = 2 \pm 2i\sqrt{2} \text{ or } x = 2 \pm i$
The solution set is $\{2 - 2i\sqrt{2}, 2 + 2i\sqrt{2}, 2 - i, 2 + i\}$.

15. $(x^2-3) - 3\sqrt{x^2-3} - 28 = 0$
Let $u = \sqrt{x^2-3}$, then $u^2 = (\sqrt{x^2-3})^2 = (x^2-3)$ and so
$u^2 - 3u - 28 = 0$
$(u-7)(u+4) = 0$
So $u = 7$ or $u = -4$. Then substitute $\sqrt{x^2-3}$ for u.
$\sqrt{x^2-3} = 7$ or $\sqrt{x^2-3} = -4$
$x^2 - 3 = 49$ $x^2 - 3 = 16$
$x^2 = 52$ $x^2 = 19$
$x = \pm\sqrt{52} = \pm2\sqrt{13}$ $x = \pm\sqrt{19}$
The solution set is $\{2\sqrt{13},-2\sqrt{13}\}$. *Note:* $\sqrt{19}$ and $-\sqrt{19}$ are extraneous solutions.

19. $y^{2/3} + 3y^{1/3} - 10 = 0$
Let $u = y^{1/3}$, then
$u^2 + 3u - 10 = 0$
$(u+5)(u-2) = 0$
$u = -5 \text{ or } u = 2$
Substitute $y^{1/3}$ for u
$y^{1/3} = -5 \text{ or } y^{1/3} = 2$
$y = -125 \text{ or } y = 8$ Cube each member.
The solution set is $\{-125,8\}$.

30. $(t^2 - t)^2 - 4(t^2 - t) - 12 = 0$
Let $u = t^2 - t$, then $u^2 = (t^2 - t)^2$ and so
$u^2 - 4u - 12 = 0$
$(u-6)(u+2) = 0$ so $u = 6$ or $u = -2$.
Then $t^2 - t = 6$
$t^2 - t - 6 = 0$
$(t-3)(t+2) = 0$
So $t = 3$ or $t = -2$.
or
$t^2 - t = -2$
$t^2 - t + 2 = 0$
$t = \dfrac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(2)}}{2(1)}$
$= \dfrac{1 \pm \sqrt{1-8}}{2} = \dfrac{1 \pm i\sqrt{7}}{2}$
$\left\{3, -2, \dfrac{1 + i\sqrt{7}}{2}, \dfrac{1 - i\sqrt{7}}{2}\right\}$

1. $\dfrac{-4x^2 - 5x}{(x+2)(x-2)}$ **2.** $\dfrac{x+4}{x-1}$ **3. a.** $y = 8$ **b.** $y = -27$

c. $y = -2$ **4.** $\left\{x \mid x \geq \dfrac{5}{2}\right\} = \left[\dfrac{5}{2}, \infty\right)$ **5.** $\{-1,4\}$

6. $\{x \mid -1 \leq x \leq 7\} = [-1,7]$

7. $\left\{x \mid x < -1 \text{ or } x > \dfrac{1}{3}\right\} = (-\infty,-1) \cup \left(\dfrac{1}{3},\infty\right)$

Exercise 6–7

Answers to odd-numbered problems

1. $\{x \mid x < -3 \text{ or } x > 1\} = (-\infty,-3) \cup (1,\infty)$

3. $\{p \mid -7 \leq p \leq -2\} = [-7,-2]$

5. $\{r \mid r \leq 0 \text{ or } r \geq 1\} = (-\infty,0] \cup [1,\infty)$

7. $\{x \mid 1 < x < 4\} = (1,4)$

9. $\{q \mid -3 \leq q \leq 6\} = [-3,6]$

11. $\{x \mid 1 < x < 2\} = (1,2)$

13. $\{u \mid u \leq -6 \text{ or } u \geq 2\} = (-\infty,-6] \cup [2,\infty)$

15. $\{y \mid y < -2 \text{ or } y > 2\} = (-\infty,-2) \cup (2,\infty)$

17. $\{x \mid -\sqrt{3} < x < \sqrt{3}\} = (-\sqrt{3},\sqrt{3})$

19. $\{x \mid 0 < x < 5\} = (0,5)$

21. $\{w \mid -3 \leq w \leq 0\} = [-3,0]$

23. $\left\{y \mid -2 < y < \dfrac{3}{2}\right\} = \left(-2,\dfrac{3}{2}\right)$

25. $\left\{w \mid w \leq -5 \text{ or } w \geq -\dfrac{1}{3}\right\} = (-\infty, -5] \cup \left[-\dfrac{1}{3}, \infty\right)$
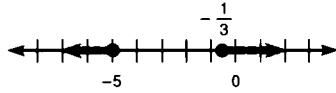
27. $\left\{v \mid -\dfrac{2}{3} < v < \dfrac{4}{3}\right\} = \left(-\dfrac{2}{3}, \dfrac{4}{3}\right)$
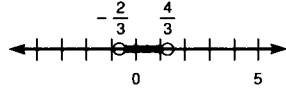
29. $\left\{x \mid x \leq -\dfrac{4}{5} \text{ or } x \geq 1\right\} = \left(-\infty, -\dfrac{4}{5}\right] \cup [1, \infty)$

31. $\{y \mid y < -6 \text{ or } -4 < y < 5\} = (-\infty, -6) \cup (-4, 5)$
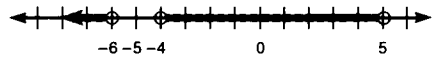
33. $\left\{t \mid -5 \leq t \leq 0 \text{ or } t \geq \dfrac{8}{3}\right\} = [-5, 0] \cup \left[\dfrac{8}{3}, \infty\right)$
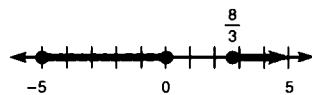
35. $\{p \mid p \leq -3 \text{ or } p \geq -1\} = (-\infty, -3] \cup [-1, \infty)$
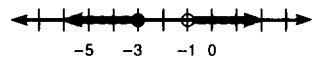
37. $\left\{t \mid -\dfrac{4}{5} < t < \dfrac{3}{7}\right\} = \left(-\dfrac{4}{5}, \dfrac{3}{7}\right)$

39. $\left\{y \mid y < 0 \text{ or } y > \dfrac{5}{3}\right\} = (-\infty, 0) \cup \left(\dfrac{5}{3}, \infty\right)$

41. $\{y \mid y \leq 1 \text{ or } y > 6\} = (-\infty, -1] \cup (6, \infty)$

43. $\left\{t \mid -4 < t < -\dfrac{8}{3}\right\} = \left(-4, -\dfrac{8}{3}\right)$
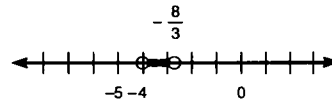
45. $\left\{x \mid -\dfrac{15}{2} < x < -5\right\} = \left(-\dfrac{15}{2}, -5\right)$
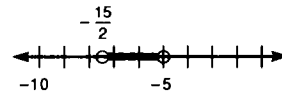
47. $\left\{x \mid x < 7 \text{ or } x \geq \dfrac{34}{3}\right\} = (-\infty, 7) \cup \left[\dfrac{34}{3}, \infty\right)$
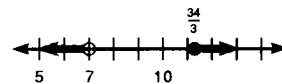
49. $\{z \mid z < -2 \text{ or } z > 7\} = (-\infty, -2) \cup (7, \infty)$
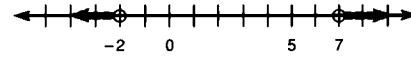
51. $\left\{q \mid q < -4 \text{ or } -\dfrac{7}{2} \leq q \leq 0\right\} = (-\infty, -4) \cup \left[-\dfrac{7}{2}, 0\right]$

53. $\left\{t \mid t < 0 \text{ or } \dfrac{1}{9} < t < \dfrac{2}{3}\right\} = (-\infty, 0) \cup \left(\dfrac{1}{9}, \dfrac{2}{3}\right)$

Solutions to trial exercise problems

14.

$$x^2 - 1 < 0$$
$$(x + 1)(x - 1) < 0$$

The critical numbers are -1 and 1.

The solution set is $\{x \mid -1 < x < 1\} = (-1, 1)$.

	$x + 1$	$x - 1$	product
$x < -1$	$-$	$-$	$+$
$-1 < x < 1$	$+$	$-$	$\ominus$
$x > 1$	$+$	$+$	$+$

16.

$$p^2 \geq 5$$
$$p^2 - 5 \geq 0$$
$$(p + \sqrt{5})(p - \sqrt{5}) \geq 0$$

The critical numbers are $-\sqrt{5}$ and $\sqrt{5}$.

The solution set is $\{p \mid p \leq -\sqrt{5} \text{ or } p \geq \sqrt{5}\} = [-\sqrt{5}, \sqrt{5}]$.
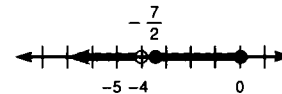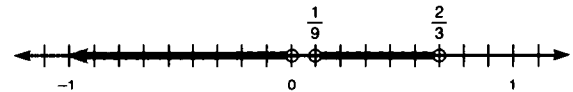
	$p + \sqrt{5}$	$p - \sqrt{5}$	product
$p < -\sqrt{5}$	$-$	$-$	$\oplus$
$-\sqrt{5} < p < \sqrt{5}$	$+$	$-$	$-$
$p > \sqrt{5}$	$+$	$+$	$\oplus$

19. $x^2 - 5x < 0$
$x(x - 5) < 0$
The critical numbers are 0 and 5.
The solution set is $\{x | 0 < x < 5\} = (0,5)$.
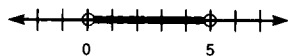
	x	$x - 5$	product
$x < 0$	$-$	$-$	$+$
$0 < x < 5$	$+$	$-$	$\ominus$
$x > 5$	$+$	$+$	$+$

22. $2x^2 - 7x - 4 > 0$
$(2x + 1)(x - 4) > 0$
The critical numbers are $-\dfrac{1}{2}$ and 4.

	$2x + 1$	$x - 4$	product
$x < -\dfrac{1}{2}$	$-$	$-$	$\oplus$
$-\dfrac{1}{2} < x < 4$	$+$	$-$	$-$
$x > 4$	$+$	$+$	$\oplus$

The solution set is $\left\{x | x < -\dfrac{1}{2} \text{ or } x > 4\right\} = \left(-\infty, -\dfrac{1}{2}\right) \cup (4,\infty)$.
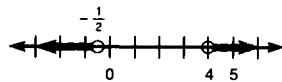
30. $(x - 3)(x + 1)(x - 2) > 0$ The critical numbers are -1, 2, and 3.

	$x - 3$	$x + 1$	$x - 2$	product
$x < -1$	$-$	$-$	$-$	$-$
$-1 < x < 2$	$-$	$+$	$-$	$\oplus$
$2 < x < 3$	$-$	$+$	$+$	$-$
$x > 3$	$+$	$+$	$+$	$\oplus$

The solution set is $\{x | -1 < x < 2 \text{ or } x > 3\} = (-1,2) \cup (3,\infty)$.
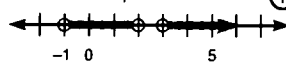
38. $\dfrac{1}{x} > 2$

(a) Set the denominator equal to zero. Then $x = 0$.
(b) Replace $>$ with $=$ and solve for x.
$$\frac{1}{x} = 2$$
$$2x = 1$$
$$x = \frac{1}{2}$$

The critical numbers are 0 and $\dfrac{1}{2}$ and the regions are $x < 0$,

$0 < x < \dfrac{1}{2}$, and $x > \dfrac{1}{2}$.

Let $x = -1$, then $\dfrac{1}{-1} = -1 > 2$ (False)

Let $x = \dfrac{1}{4}$, then $\dfrac{1}{\frac{1}{4}} = 4 > 2$ (True)

Let $x = 1$, then $\dfrac{1}{1} = 1 > 2$ (False)

The solution set is $\left\{x | 0 < x < \dfrac{1}{2}\right\} = \left(0, \dfrac{1}{2}\right)$.
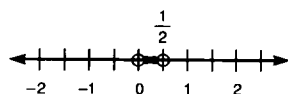
45. $\dfrac{x}{x + 5} > 3$

(a) Set $x + 5 = 0$
$$x = -5.$$
(b) Replace $>$ with $=$.
$$\frac{x}{x + 5} = 3$$
$$3x + 15 = x$$
$$2x = -15$$
$$x = -\frac{15}{2}$$

The critical numbers are $-\dfrac{15}{2}$ and -5 and the regions are

$x < -\dfrac{15}{2}, \quad -\dfrac{15}{2} < x < -5, \quad x > -5$.

Let $x = -8$, then $\dfrac{-8}{-8 + 5} = \dfrac{8}{3} > 3$ (False)

Let $x = -6$, then $\dfrac{-6}{-6 + 5} = 6 > 3$ (True)

Let $x = -4$, then $\dfrac{-4}{-4 + 5} = -4 > 3$ (False)

The solution set is $\left\{x \left| -\dfrac{15}{2} < x < -5\right.\right\} = \left(-\dfrac{15}{2}, -5\right)$.
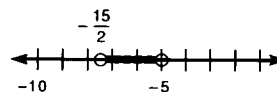
50. $\dfrac{2p}{p-2} \le p$ *(Note: $p \ne 2$)*

    (a) Set $p - 2 = 0$, $p = 2$.

    (b) Replace $\le$ with $=$.

$$\frac{2p}{p-2} = p$$
$$p^2 - 2p = 2p$$
$$p^2 - 4p = 0$$
$$p(p-4) = 0$$
$$p = 0 \text{ or } p = 4$$

The critical numbers are 0, 2, and 4 so the regions are $p < 0$, $0 < p < 2$, $2 < p < 4$, and $p > 4$.

Let $p = -1$, then $\dfrac{2(-1)}{-1-2} = \dfrac{2}{3} \le -1$ (False)

Let $p = 1$, then $\dfrac{2(1)}{1-2} = -2 \le 1$ (True)

Let $p = 3$, then $\dfrac{2(3)}{3-2} = 6 \le 3$ (False)

Let $p = 5$, then $\dfrac{2(5)}{5-2} = \dfrac{10}{3} \le 5$ (True)

The solution set is $\{p\,|\,0 \le p < 2 \text{ or } p \ge 4\} = [0,2) \cup [4,\infty)$.
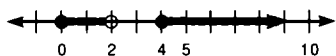
Review exercises

1. $r = \sqrt{\dfrac{A}{2\pi}} = \dfrac{\sqrt{2\pi A}}{2\pi}$ **2.** $x - 2$ **3.** $\{1,2\}$ **4. a.** $y = 17$

b. $y = -11$ **c.** $y = -3$ **5.** $-\dfrac{5}{3}$ **6.** $\ell = 29$ m, $w = 26$ m

Chapter 6 review

1. $\{10,-1\}$ **2.** $\{8,-4\}$ **3.** $\left\{0,\dfrac{7}{4}\right\}$ **4.** $\{-2,2\}$

5. $\left\{\dfrac{1}{2},\dfrac{5}{2}\right\}$ **6.** $\left\{\dfrac{5}{2},-1\right\}$ **7.** $\left\{\dfrac{1}{3},2\right\}$ **8.** $\{-3,1\}$

9. $\{1,-15\}$ **10.** $\left\{0,\dfrac{8}{3}\right\}$ **11.** $20 \pm 10\sqrt{3} \approx 2.7$ sec or

37.3 sec; 40 sec   **12.** $\left\{\dfrac{-3-\sqrt{41}}{2},\dfrac{-3+\sqrt{41}}{2}\right\}$ **13.** $\left\{\dfrac{1}{2},1\right\}$

14. $\left\{\dfrac{-3-i\sqrt{11}}{10},\dfrac{-3+i\sqrt{11}}{10}\right\}$ **15.** $\left\{\dfrac{1-\sqrt{85}}{14},\dfrac{1+\sqrt{85}}{14}\right\}$

16. $\left\{\dfrac{3-5i\sqrt{15}}{16},\dfrac{3+5i\sqrt{15}}{16}\right\}$ **17.** $\{10,1\}$

18. $\left\{\dfrac{-3-\sqrt{33}}{6},\dfrac{-3+\sqrt{33}}{6}\right\}$ **19.** $\{-\sqrt{2},\sqrt{2}\}$

20. $\left\{\dfrac{-1-i\sqrt{31}}{4},\dfrac{-1+i\sqrt{31}}{4}\right\}$ **21.** $\left\{-1,\dfrac{5}{3}\right\}$

22. $\left\{\dfrac{11}{4},1\right\}$ **23.** $x = \dfrac{-y \pm y\sqrt{33}}{8}$ **24.** $x = L$ or $x = \dfrac{1}{2}L$

25. $\dfrac{15 \pm 5\sqrt{6}}{4} \approx 0.7$ sec or 6.8 sec   **26.** 200 cm by 50 cm or 2 m

by $\dfrac{1}{2}$ m   **27.** 15 ft, 8 ft, 17 ft   **28.** Mary, $11 + \sqrt{145} \approx 23$ hr;

Dick, $13 + \sqrt{145} \approx 25$ hr   **29.** $\{5\}$; -4 is extraneous

30. $\left\{\dfrac{4}{3}\right\}$; $-\dfrac{1}{3}$ is extraneous   **31.** $\left\{\dfrac{1}{4}\right\}$ **32.** $\{3,-2\}$ **33.** $\{5\}$;

$\dfrac{5}{4}$ is extraneous   **34.** $V = \dfrac{4\pi r^3}{3}$ **35.** $\{\sqrt{7},-\sqrt{7},i\sqrt{2},-i\sqrt{2}\}$

36. $\{0,4\}$ **37.** $\emptyset$; 1 and 64 are extraneous roots

38. $\{4\}$; $\dfrac{1}{9}$ is extraneous root   **39.** $\left[-8,\dfrac{27}{8}\right]$ **40.** $\left\{-1,\dfrac{1}{11}\right\}$

41. $\left\{-\dfrac{\sqrt{2}}{2},\dfrac{\sqrt{2}}{2},\dfrac{-i\sqrt{10}}{2},\dfrac{i\sqrt{10}}{2}\right\}$

42. $\{x\,|\,x < -7 \text{ or } x > 3\} = (-\infty,-7) \cup (3,\infty)$
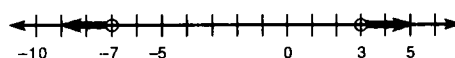
43. $\left\{x\,\middle|\,-\dfrac{4}{3} \le x \le \dfrac{1}{2}\right\} = \left[-\dfrac{4}{3},\dfrac{1}{2}\right]$
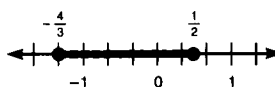
44. $\left\{y\,\middle|\,y \le 0 \text{ or } y \ge \dfrac{3}{2}\right\} = (-\infty,0] \cup \left[\dfrac{3}{2},0\right)$
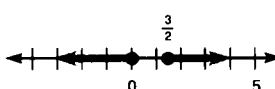
45. $\left\{z\,\middle|\,-\dfrac{2}{3} < z < \dfrac{2}{3}\right\} = \left(-\dfrac{2}{3},\dfrac{2}{3}\right)$
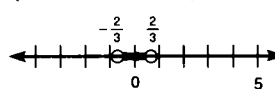
46. $\{m\,|\,m < -1 \text{ or } m > 7\} = (-\infty,-1) \cup (7,\infty)$
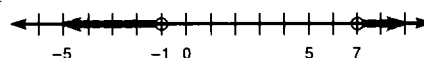
47. $\left\{p\,\middle|\,-\dfrac{3}{4} < p < 2\right\} = \left(-\dfrac{3}{4},2\right)$
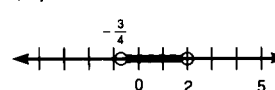
48. $\{x\,|\,x \le -3 \text{ or } x \ge 3\} = (-\infty,-3] \cup [3,\infty)$
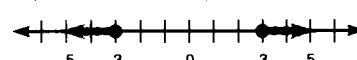
49. $\{y\,|\,-2 \le y \le 16\} = [-2,16]$
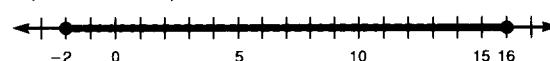
50. $\{y\,|\,y < -4 \text{ or } 2 < y < 5\} = (-\infty,-4) \cup (2,5)$
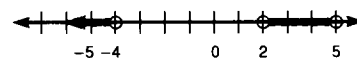
51. $\{m\,|\,m \le -3 \text{ or } m > 1\} = (-\infty,-3] \cup (1,\infty)$
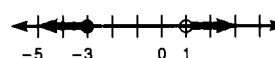
52. $\{x\,|\,x \le -17 \text{ or } x > -7\} = (-\infty,-17] \cup (-7,\infty)$
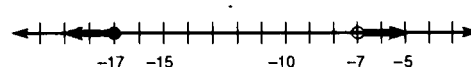
Chapter 6 cumulative test

1. -12 2. 2 3. 56 4. $7xy^2 - 6xy + 6x^2y$
5. $9x^2 + 12xy + 4y^2$ 6. $16y^2 - 1$ 7. $9x^3 - 16x^2 - 8$

8. $-3,375x^6y^9$ 9. a^{20} 10. $\dfrac{-2b^5}{a^5}$ 11. (a) 6, (b) 1, (c) 41

12. -12 13. $\dfrac{2(a + 1)}{a - 4}$ 14. $\dfrac{3}{4}$ 15. $\dfrac{x + 1}{x - 3}$

16. $\dfrac{2x^2 - 3x + 1}{x^2 + x - 42}$ 17. $\dfrac{12p - 3}{(p - 9)(p + 2)(p - 2)}$ 18. $\dfrac{a + 5}{a - 6}$

19. $\left\{-\dfrac{1}{2}, -3\right\}$ 20. $\{x \mid 0 \le x \le 5\} = [0,5]$

21. $\left\{x \mid x < \dfrac{3}{2} \text{ or } x > \dfrac{11}{2}\right\} = \left(-\infty, \dfrac{3}{2}\right) \cup \left(\dfrac{11}{2}, \infty\right)$ 22. $\{-7\}$

23. $\{x \mid x \ge -33\} = [-33, \infty)$ 24. $\left\{-\dfrac{5}{3}\right\}$ 25. $w = \dfrac{P - 2\ell}{2}$

26. $\dfrac{10y - 4}{4y + 5}$ 27. $6\sqrt{3} - 3$ 28. $16\sqrt{3}$ 29. $-3\sqrt[3]{3}$ 30. 13

31. $47 - 12\sqrt{15}$ 32. $\dfrac{4\sqrt{5}}{5}$ 33. $2\sqrt{6} - 5$ 34. $\dfrac{-5 + 2i}{29}$ or

$\dfrac{-5}{29} + \dfrac{2}{29}i$ 35. $\{14,1\}$ 36. $\left\{\dfrac{1 + i\sqrt{59}}{10}, \dfrac{1 - i\sqrt{59}}{10}\right\}$

37. $\left\{\dfrac{1 - i\sqrt{35}}{6}, \dfrac{1 + i\sqrt{35}}{6}\right\}$ 38. $\{z \mid -1 \le z \le 3\} = [-1,3]$

39. $\{i\sqrt{5}, -i\sqrt{5}, \sqrt{10}, -\sqrt{10}\}$ 40. $\{y \mid -7 < y < 10\} = (-7,10)$

41. $4x^4 - 4x^3 + x^2 - 1 + \dfrac{2}{x + 1}$

Chapter 7

Exercise 7–1

Answers to odd-numbered problems

1. $(2,4)$; quadrant I (see graph)
3. $(-4,3)$; quadrant II (see graph)
5. $(-1,-3)$; quadrant III (see graph)
7. $(4,0)$; quadrantal (see graph)

9. $(0,-1)$; quadrantal (see graph)
11. $\left(\dfrac{1}{2}, 3\right)$; quadrant I (see graph)
13. $\left(-\dfrac{7}{2}, -\dfrac{5}{2}\right)$; quadrant III (see graph)

15. $(-2,7), (0,3), (2,-1), \left(\dfrac{3}{2}, 0\right)$;

$y = -2x + 3$
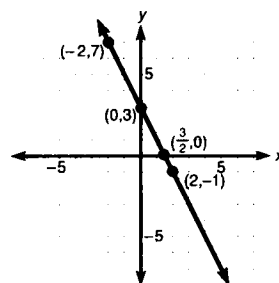
17. $(-5,9), (-3,7), (0,4), (4,0)$;

$y = -x + 4$
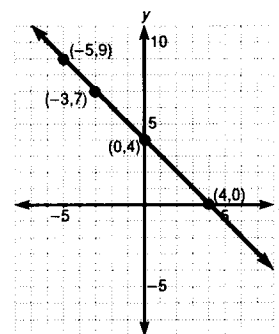
19. $(-3,-5), (0,-2), (2,0), (2,0)$;

$y = x - 2$

21. $\left(-1, -\dfrac{2}{3}\right), \left(0, -\dfrac{1}{3}\right), (1,0), (1,0)$;

$y = \dfrac{1}{3}x - \dfrac{1}{3}$

Contents

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